Answers

Chapter 1

Lesson 1.1, page 12

1. e.g., The manager made the conjecture that each type of ski would sell equally as well as the others.

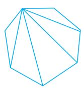
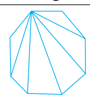
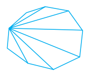
2. Tomas's conjecture is not reasonable. $99(11) = 1089$

3. e.g., The sum of two even integers is always even. For example, $6 + 12 = 18 \quad 34 + 72 = 106$

4. e.g., The yellow symbolizes the wheat fields of Saskatchewan, the green symbolizes the northern forests, and the fleur de lys represents la Francophonie.

5. e.g., Mary made the conjecture that the sum of the angles in quadrilaterals is 360°.

6. e.g., The fewest number of triangles in a polygon is the number of sides subtracted by 2.

Polygon	heptagon	octagon	nonagon
Fewest Number of Triangles	 5	 6	 7

7. e.g., The result is always an even number ending with a decimal of .25.

8. **a)** e.g., The sums of the digits of multiples of 3 are always 3, 6, or 9.

9. e.g., The sum of one odd integer and one even integer is always odd.
$3 + 4 = 7$
$-11 + 44 = 33$
$90 + 121 = 211$

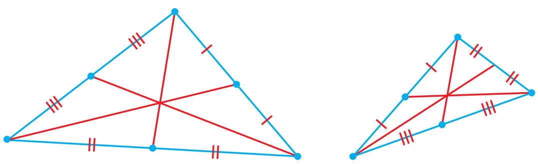
10. e.g., The temperature on November 1 in Hay River never goes above 5 °C. My conjecture is supported by the data: none of the temperatures are above 5 °C.

11. e.g., Paula's conjecture is reasonable. When you multiply an odd digit with an odd digit, the result is odd:
$1(1) = 1; 3(3) = 9; 5(5) = 25; 7(7) = 49; 9(9) = 81$
Since the ones of a product are the result of a multiplication of two digits, squaring an odd integer will always result in an odd integer.

12. e.g., The diagonals of rectangles intersect each other at their midpoints. I used my ruler to check various rectangles.

13. e.g., Text messages are written using small keypads or keyboards, making text entry difficult. Abbreviations reduce the difficult typing that needs to be done, e.g., LOL is 3 characters, "laugh out loud" is 14.

14. e.g., Nick made the conjecture that the medians of a triangle always intersect at one point.

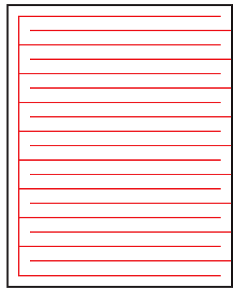


15. e.g., If March comes in like a lamb, it will go out like a lion. People may have noticed that when the weather was mild at the beginning of March, or near the end of winter, there would be bad weather at the end of March, or near the beginning of spring.

16. e.g., The town will be in the bottom right of the map near the mouth of the large river. People tend to live near bodies of fresh water, and this is one of the few flat areas on the map.

17. e.g., If social networking sites were the only way to pass information among people, it is reasonable that everyone would access such a site once per day to connect with people or obtain news. Because of various schedules (e.g., working or sleeping during the day, time zones), it is reasonable that it would take at least 12 h for the news to reach the whole Canadian population.

18. e.g., Thérèse's conjecture is possible. Cut the paper along the red lines. Unfold to form a hole larger than the original piece of paper.



19. e.g., A conjecture is a belief, and inferences and hypotheses are also beliefs. However, conjectures, inferences, and hypotheses are validated differently because they relate to different subjects: mathematics/logic, literature, and science.

20. e.g., The photograph is of a shadow of a statue holding a globe.
The photograph is of a shadow of a soccer goalie, near the goal, holding the ball above her head.
The picture is of a shadow of a child holding a ball above his head near a swing set.

21. e.g., The statement is not a conjecture. The company making the claim probably surveyed some dentists to get their opinion; however, these dentists' opinion may not represent that of all dentists.

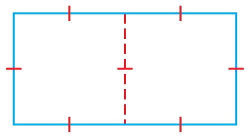
22. e.g., Conjectures about sports may not be accurate because a player or a team's performance may change depending on the health of the player or the constitution of the team.

Lesson 1.2, page 17

1. e.g., The dimensions of the tabletops are the same. A ruler may be used to measure them.

2. e.g., The pattern will continue until 12345678987654321; after that, it will change. I can test my conjecture using a spreadsheet.

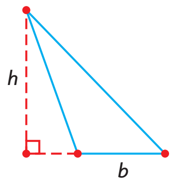
3. e.g., When two regular congruent polygons are positioned so that there is a common side, the polygon formed will have $2n - 2$ sides, where n is the number of sides in one original polygon. My conjecture is invalid. The resulting figure is 4-sided:



Lesson 1.3, page 22

1. e.g.,
a) 0 is a number that is not negative, and is not positive.
b) 2 is a prime number that is not odd.
c) Muggsy Bogues was an NBA player who was 1.6 m (5 ft 3 in.) tall.

d) The height of a triangle can lie outside the triangle.



e) If a city's shape is roughly rectangular and it lies along a northeast-southwest axis, then the map will be set to accommodate the city's shape, and the north arrow would instead point toward a corner of the map.

f) $\sqrt{0.01} = 0.1$

g) $-10 + 5 = -5$

h) Travelling north in the southern hemisphere generally results in a warmer climate.

2. Disagree. e.g., The sides of a rhombus are equal, but its angles may not be 90°.

3. Disagree. e.g., $1(10) = 10$

4. Disagree. e.g., $9 + 12 = 21$

5. Disagree. e.g., 99. $9 + 9 = 18$. $18 \neq 9$

6. Disagree. e.g., a kite with angles of 90°, 45°, 90°, and 135°

7. e.g., Claire's conjecture seems reasonable because so many combinations are possible. I tried a few examples.

Number	Expression
6	$\frac{6}{\sqrt{4}}(7 - 5)$
10	$\frac{6(5)}{7 - 4}$
19	$4(5) - 7 + 6$

8. e.g., My evidence strengthens George's conjecture. For example,
$123456789 \cdot 4 + 9 = 493827165$
$1234567891011 \cdot 4 + 11 = 4938271564055$

9. e.g., The sum of digits in any multiple of 9 greater than zero will be divisible by 9.

10. e.g., Patrice's conjecture is reasonable. Integers separated by a value of 2 will both be odd or both be even, and their squares will both be odd or both be even. Adding two even numbers together and adding two odd numbers together result in an even number.

11. e.g., Geoff's conjecture is not valid. Kites and rhombuses have perpendicular diagonals.

12. e.g., Amy's conjecture could be changed to "When any number greater than 1 is multiplied by itself, it will be greater than the starting number."

13. e.g., Any real number is divisible by another real number.
$\frac{425.353}{1.35} = 315.076...$ $\frac{\pi}{\sqrt[3]{9}} = 1.510...$
Counterexample: 0 is a real number for which division is not defined.

14. Disagree. e.g., The number 2 cannot be written as the sum of consecutive numbers.

15. e.g., Blake's claim is not valid. The number 3 cannot be written as the sum of three primes.

16. e.g.,

a) $18 = 5 + 13$
$54 = 11 + 43$
$106 = 5 + 101$

b) A counterexample would be an even number that is not equal to the sum of two primes.

17. e.g.,

a) The number picked and the final result are the same.

b) I cannot find a counterexample. This does not imply that the conjecture is valid, but it does strengthen it.

18. e.g., Inductive reasoning can be used to make a conjecture; a conjecture is supported by evidence and can be invalidated by a counterexample.

19. Disagree. e.g., $4^2 - 3 = 13$

20. **a)** e.g.,

0%	There won't be rain, even if it's cloudy. Perfect for a day at the lake.
10%	Little chance of rain or snow. A good day for a hike.
20%	No rain is expected; good weather for soccer.
30%	There's a small chance of rain. I'll risk a game of ultimate at the local park.
40%	It might rain, but I might skateboard close to home.
50%	It's a good idea to bring an umbrella or rain jacket on the way to school.
60%	It's a very good idea to bring an umbrella or rain jacket on the way to school.
70%	The chance for no rain is 3 out of 10—I'll stay inside and watch a movie.
80%	Rain is likely. I'll read a book.
90%	It will almost certainly rain. I'll spend time surfing the Internet.
100%	It will definitely rain. I'll play a game of basketball indoors.

b) e.g., For probabilities of 30% or less, I'd definitely go outside because of the low chance of rain. For probabilities of 40%–50%, I would still go outdoors because it's about a 1 in 2 chance it will rain. For 60% or more, which is closer to a two-thirds chance that it will rain, I would definitely stay indoors.

21. Agree. e.g., If n is odd, its square will be odd. Two odd numbers and one even number added together result in an even number. If n is even, then three even numbers are added together, and that results in an even number.

Lesson 1.4, page 31

1. e.g., Let n be any number.
$(n - 3) + (n - 2) + (n - 1) + n + (n + 1)$
$+ (n + 2) + (n + 3) = 7n$
Since n is the median, Chuck's conjecture is true.

2. Austin got a good haircut.

3. e.g., The angles formed at the intersection of the diagonals are two pairs of opposite, equal angles.

4. e.g., Let $2n$ and $2m$ represent any two even numbers.
$2n + 2m = 2(n + m)$
Since 2 is a factor of the sum, the sum is even.

5. Let $2n + 1$ represent an odd number and $2m$ represent an even number.
$2m(2n + 1) = 4mn + 2m$
$2m(2n + 1) = 2(2mn + m)$
Since 2 is a factor of the product, the product is even.

6. e.g., Using the Pythagorean theorem, we can show that the first and third triangles have a right angle opposite the hypotenuse.
$4^2 + 3^2 = 16 + 9$ $6^2 + 8^2 = 36 + 64$
$4^2 + 3^2 = 25$ $6^2 + 8^2 = 100$
$4^2 + 3^2 = 5^2$ $6^2 + 8^2 = 10^2$
So, angles a and c are 90°.
Angle b and the right angle are supplementary. Angle b is 90°.
Since angles a, b, and c are all 90°, they are equal to each other.

7. **a)** e.g.,

n	5	0	− 11
× 4	20	0	− 44
+ 10	30	10	− 34
÷ 2	15	5	− 17
− 5	10	0	− 22
÷ 2	5	0	− 11
+ 3	8	3	− 8

b)

n	n
× 4	$4n$
+ 10	$4n + 10$
÷ 2	$2n + 5$
− 5	$2n$
÷ 2	n
+ 3	$n + 3$

8. e.g., The premises do not exclude other pants from being expensive.

9. e.g.,

n	n
× 2	$2n$
+ 6	$2n + 6$
× 2	$4n + 12$
− 4	$4n + 8$
÷ 4	$n + 2$
− 2	n

10. e.g., Let $2n + 1$ represent any odd integer.
$(2n + 1)^2 = 4n^2 + 2n + 2n + 1$
The numbers $4n^2$ and $2n$ are even. The addition of 1 makes the result odd.

11. e.g.,
$4^2 - 6^2 = 16 - 36$ $\quad$ $5^2 - 7^2 = 25 - 49$
$4^2 - 6^2 = -20$ $\quad$ $5^2 - 7^2 = -24$
Let n represent any number.
$n^2 - (n - 2)^2 = n^2 - (n^2 - 4n + 4)$
$n^2 - (n - 2)^2 = n^2 - n^2 + 4n - 4$
$n^2 - (n - 2)^2 = 4n - 4$
$n^2 - (n - 2)^2 = 4(n - 1)$
The difference is a multiple of 4.

12. e.g.,

Choose a number.	n
Add 5.	$n + 5$
Multiply by 3.	$3n + 15$
Add 3.	$3n + 18$
Divide by 3.	$n + 6$
Subtract the number you started with.	6

13. e.g., Let $abcd$ represent any four-digit number.
$abcd = 1000a + 100b + 10c + d$
$abcd = 2(500a + 50b + 5c) + d$
The number $abcd$ is divisible by 2 only when d is divisible by 2.

14. e.g., Let ab represent any two-digit number.
$ab = 10a + b$
$ab = 5(2a) + b$
The number ab is divisible by 5 only when b is divisible by 5.
Let abc represent any three-digit number.
$abc = 100a + 10b + c$
$abc = 5(20a + 2b) + c$
The number abc is divisible by 5 only when c is divisible by 5.

15. e.g., Let ab represent any two-digit number.
$ab = 10a + b$
$ab = 9a + (a + b)$
The number ab is divisible by 9 only when $(a + b)$ is divisible by 9.
Let abc represent any three-digit number.
$abc = 100a + 10b + c$
$abc = 99a + 9b + (a + b + c)$
The number abc is divisible by 9 only when $(a + b + c)$ is divisible by 9.

16. e.g.,
$\frac{5^2}{4} = 6.25$ $\quad$ $\frac{11^2}{4} = 30.25$ $\quad$ $\frac{23^2}{4} = 132.25$
When an odd number is squared and divided by four, it will always result in a decimal number ending with 0.25.
Let $2n + 1$ represent any odd number.
$$\frac{(2n + 1)^2}{4} = \frac{4n^2 + 4n + 1}{4}$$
$$\frac{(2n + 1)^2}{4} = \frac{4(n^2 + n) + 1}{4}$$
$$\frac{(2n + 1)^2}{4} = (n^2 + n) + \frac{1}{4}$$
$$\frac{(2n + 1)^2}{4} = (n^2 + n) + 0.25$$
An odd number squared, then divided by four, will always result in a decimal number ending with 0.25.

17. e.g., Joan and Garnet used inductive reasoning to provide more evidence for the conjecture, but their solutions aren't mathematical proofs. Jamie used deductive reasoning to develop a generalization that proves Simon's conjecture.

18. e.g., Let x represent the original number; let d represent the difference between x and its nearest lower multiple of 10.
Step 1: $x - d$
Step 2: $x + d$
Step 3: $(x + d)(x - d) = x^2 - d^2$
Step 4: $x^2 - d^2 + d^2 = x^2$

19. e.g., $n^2 + n + 2 = n(n + 1) + 2$
The expression $n(n + 1)$ represents the product of an odd integer and an even integer. The product of an odd integer and an even integer is always even (see question 5). Adding 2 to an even number results in an even number.

20. e.g., Conjecture: The product of two consecutive natural numbers is always even.
The product of two consecutive natural numbers is the product of an odd integer and an even integer.

Let $2n$ and $2n + 1$ represent any two consecutive natural numbers when the even number is less than the odd number.
$2n(2n + 1) = 4n^2 + 2n$
$2n(2n + 1) = 2(2n^2 + n)$
Let $2n$ and $2n - 1$ represent any two consecutive natural numbers when the odd number is less than the even number.
$2n(2n - 1) = 4n^2 - 2n$
$2n(2n - 1) = 2(2n^2 - n)$
In both cases, the product has a factor of 2. The product of two consecutive natural numbers is always even.

Mid-Chapter Review, page 35

1. e.g., The medicine wheel's spokes may have pointed toward celestial bodies at solstices and equinoxes.

2. e.g., The squares follow a pattern of $t + 1$ fours, t eights, and 1 nine, where t is the term number. For example, the second term, $t = 2$, is $667^2 = 444889$
The 25th term in the pattern will be 25 sixes and 1 seven, squared, and the result will be 26 fours, 25 eights, and 1 nine.

3. e.g.,
a) The sum of the numbers in the 10th row will be 512.
b) The sum of any row is $2^{(r-1)}$, where r is the row number.

4. e.g., Glenda's conjecture seems reasonable. For the countries whose names begin with A, B, C, or S, there are 30 countries whose names end with a consonant and 42 whose names end with a vowel.

5. e.g., Igor Larionov is a Russian hockey player who was inducted into the Hockey Hall of Fame in 2008.

6. Disagree. e.g., The diagonals of parallelograms and rhombuses also bisect each other.

7. Disagree. e.g., For example, the conjecture "all prime numbers are odd" can be supported by 10 examples (3, 5, 7, 9, 11, 13, 17, 19, 23, 29), but the conjecture is not valid: 2 is an even prime number.

8. e.g.,
a) If 5 is chosen, the result is 5. If 2 is chosen, the result is 5.
Conjecture: The number trick always has a result of 5.

n	n
+ 3	$n + 3$
× 2	$2n + 6$
+ 4	$2n + 10$
÷ 2	$n + 5$
− n	5

b) If 7 is chosen, the result is 7. If 4 is chosen, the result is 7.
Conjecture: The number trick always has a result of 7.

n	n
× 2	$2n$
+ 9	$2n + 9$
+ n	$3n + 9$
÷ 3	$n + 3$
+ 4	$n + 7$
− n	7

9. e.g.,
Let n, $n + 1$, $n + 2$, and $n + 3$ represent any four consecutive natural numbers.
$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$
$n + (n + 1) + (n + 2) + (n + 3) = 2(2n + 3)$
Since 2 is a factor of the sum, the sum of four consecutive natural numbers is always even.

10. e.g.,

$(7 + 11)^2 = 324$	$(1 + 10)^2 = 121$	$(3 + 5)^2 = 64$
$7^2 + 11^2 = 170$	$1^2 + 10^2 = 101$	$3^2 + 5^2 = 34$
$(7 + 11)^2 > 7^2 + 11^2$	$(1 + 10)^2 > 1^2 + 10^2$	$(3 + 5)^2 > 3^2 + 5^2$

Let n and m be any two positive integers.
The square of the sum of two positive integers:
$(n + m)^2 = n^2 + 2mn + m^2$
The sum of the squares of two positive integers:
$n^2 + m^2$
Since $2mn > 0$ for all positive integers,
$n^2 + 2mn + m^2 > n^2 + m^2$
The square of the sum of two positive integers is greater than the sum of the squares of the same two integers.

11. e.g.,
Let $2n + 1$ represent any odd integer.
$(2n + 1)^2 - (2n + 1) = (4n^2 + 4n + 1) - (2n + 1)$
$(2n + 1)^2 - (2n + 1) = 4n^2 + 2n$
$(2n + 1)^2 - (2n + 1) = 2(2n^2 + n)$
Since the difference has a factor of 2, the difference between the square of an odd integer and the integer itself is always even.

Lesson 1.5, page 42

1. e.g.,
a) The statement "all runners train on a daily basis" is invalid.
b) The reasoning leading to the conclusion is invalid. Rectangles also have four right angles.

2. e.g., The first line of the proof is invalid.

3. e.g., In line 5, Mickey divides by $(a - b)$, which is invalid because $a - b = 0$.

4. e.g., Noreen's proof is not valid. Neither figure is a triangle, as in each case what appears to be the hypotenuse is actually two segments not along the same line (determine the slope of the hypotenuse of each small triangle to verify). When no pieces overlap, the total area is the sum of the areas of the individual pieces. That total area is 32 square units.

5. Ali did not correctly divide by 2 in line 4.

6. **a)** e.g., With a street address of 630 and an age of 16:

630	630
× 2	1 260
+ 7	1 267
× 50	63 350
+ 16	63 366
− 365	63 001
+ 15	63 016

b) Connie subtracted the wrong number for days of the year. There are 365 days in the year. Her final expression should be $100n + 350 + a - 365 + 15 = 100n + a$
c) The number of the street address is multiplied by 100, making the tens and ones columns 0. The age can be added in without any values being carried.

7. e.g., In line 7, there is a division by 0. Since $a = b$, $a^2 - ab = 0$.

8. e.g., False proofs appear true because each mathematical step involved in the reasoning seems sound. In a false proof, there is one (or more) incorrect steps that are misinterpreted as being correct.

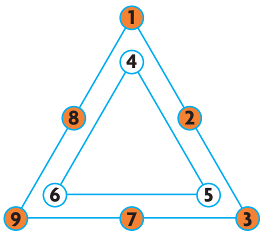
9. e.g., In general, strips of paper have two sides, a back and a front. A mark made on the front will not continue to the back unless the paper is turned over. When joined as described in the question, the piece of paper has only one side and is called a Mobius strip. A single, continuous mark can be made along the paper without turning it over.

10. e.g., The question is misleading. Each person initially paid $10 for the meal, but got $1 back. So, each person paid $9 for the meal. The meal cost $25. The waiter kept $2.
$3(9) - 2 = 25$

Lesson 1.6, page 48

1. **a)** inductive **b)** deductive **c)** inductive **d)** deductive **e)** inductive

2. e.g., Many solutions are possible. The middle triangle must add up to 15 (e.g., 1, 5, 9; 3, 4, 8) and the outer triangle must add up to 30 (e.g., 2, 3, 4, 6, 7, 8; 1, 2, 5, 6, 7, 9).



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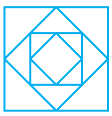


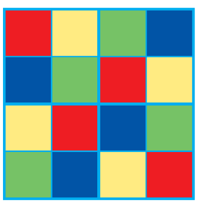
4. **a)** e.g.,

1	2	3
333	666	999
333	666	999
333	666	999
+333	+666	+999
1333	2666	3999

b) three

5. e.g.,

a)



b) Different approaches to the problem could include deductive reasoning or trial and error.

6. e.g., Let A represent one side of the river and B the other. Move goat to B; return to A. Move wolf to B; return with goat to A. Move hay to B; return. Move goat to B.

7. 28

8. The brother is a liar.

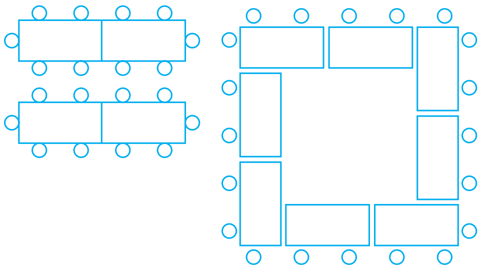
9. Bob is the quarterback, Kurt is the receiver, and Morty is the kicker.

10. e.g.,
a) The pair 2, 6 cannot be in envelope 8 because the 6 is required for envelope 13 or 14.
b) deductive

11. $abcd = 2178$

12. e.g.,

a)



b) The solution is simple and allows for everyone to be heard.

13. Tamara

14. 35

15. **a)** Suganthy
b) deductive

16. Pour water from the second pail into the fifth one.

17. e.g., A problem can be solved using inductive reasoning if it has a pattern that can be continued. A problem can be solved using deductive reasoning if general rules can be applied to obtain the solution. It is not always possible to tell which kind of reasoning is needed to solve a problem.

18. 10 days

19. Arlene

20. Pick a fruit from the apples and oranges box. Because the label is incorrect, the fruit picked determines which label goes on this box: apple or orange. Say an orange was picked. Since the labels are incorrect on the two remaining boxes, the box with the apples label is the apples and oranges box, and the box that had the oranges label on it is the apple box.

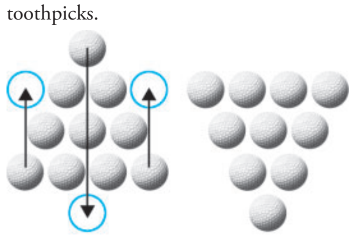
Lesson 1.7, page 55

1. 120; the pattern is $n(n + 2)$

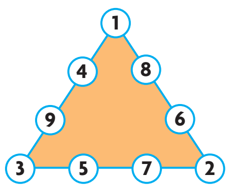
2. e.g., triple 20, double 3; double 20, double 10, double 3; triple 10, triple 10, double 3

3. e.g., To win, you must leave your opponent with 20, 16, 12, 8, and 4 toothpicks.

4.



5. **a)** e.g.,



b) e.g., I determined the possible combinations for 9, 8, and 7. I identified common addends and put those in the triangle's corners and completed the sides.

6. e.g.,

a) Numbers in each column go up by 3. Numbers in each row go up by 4.

b)

6	10	14
9	13	17
12	16	20

Selva's observation that the magic sum is three times the number in the middle square holds. My magic sum is 39.

c) The numbers in any square follow the pattern below.

$n - 7$	$n - 3$	$n + 1$
$n - 4$	n	$n + 4$
$n - 1$	$n + 3$	$n + 7$

If $n - 7$ is chosen, n and $n + 7$ may be chosen, or $n + 3$ and $n + 4$ may be chosen. All possible choices are listed below.

$(n - 7) + n + (n + 7) = 3n$

$(n - 7) + (n + 3) + (n + 4) = 3n$

$(n - 4) + (n - 3) + (n + 7) = 3n$

$(n - 4) + (n + 3) + (n + 1) = 3n$

$(n - 1) + (n - 3) + (n + 4) = 3n$

$(n - 1) + n + (n + 1) = 3n$

All choices result in the magic sum, which is three times the number in the middle square.

7. e.g.,

a) 3 (or 24 for all permutations)

b) The number in the middle is always odd.

c) Show that the number in the middle must be odd and that there are eight solutions for each odd number in the middle.

8. e.g., Put the two coins on the same diagonal.

9. Player O started the game.

10. a)

5	1	3	2	6	4
2	6	4	5	1	3
1	5	2	3	4	6
3	4	6	1	5	2
6	3	5	4	2	1
4	2	1	6	3	5

b)

6	3	4	8	2	5	7	9	1
9	5	8	3	7	1	4	6	2
2	7	1	4	6	9	3	5	8
1	4	5	6	8	3	2	7	9
3	6	7	9	1	2	8	4	5
8	9	2	7	5	4	6	1	3
7	1	3	2	9	6	5	8	4
4	8	9	5	3	7	1	2	6
5	2	6	1	4	8	9	3	7

11. a)

2	7	6
9	5	1
4	3	8

b)

2	9	4
7	5	3
6	1	8

12. 20

13.

30 × **6**	**5**	36 × **3**	2 ÷ **2**	**1**	18 + **4**
3 + **1**	**2**	**6**	7 + **4**	**3**	**5**
2	20 × **4**	**5**	5 − **1**	**6**	**3**
1 − **5**	2 − **3**	**1**	**6**	13 + **4**	**2**
4	7 + **1**	**2**	2 − **3**	**5**	**6**
2 ÷ **3**	**6**	**4**	3 − **5**	**2**	**1**

14. e.g., Using inductive reasoning, I can observe a pattern and use it to determine a solution. Using deductive reasoning, I can apply logical rules to help me solve a puzzle or determine a winning strategy for a game.

15. e.g.,

a) I would play in a spot with the fewest possibilities for placing three of my markers in a row.

b) Inductive reasoning helps me guess where my opponent will play; deductive reasoning helps me determine where I should play.

Chapter Self-Test, page 58

1. e.g.,

a) Figure 4 would have one additional cube at the end of each arm, requiring 16 cubes in all. Figure 5 would have 5 cubes more than Figure 4, with one at the end of each arm, requiring 21 cubes in all.

b) The nth structure would require $5n - 4$ cubes to build it.

c) 121 cubes

2. His conjecture isn't reasonable: the chance of the coin coming up heads is 50%.
3. e.g., A pentagon with sides of length 2 has a perimeter of 10.
4. Let $2n + 1$ and $2n + 3$ represent two consecutive odd integers. Let P represent the product of these integers.
   $P = (2n + 1)(2n + 3)$
   $P = 4n^2 + 8n + 3$
   $P = 2(2n^2 + 4n) + 3$
   $2(2n^2 + 4n)$ is an even integer, 3 is an odd integer, and the sum of any even and odd integer is an odd integer, so the product of any two consecutive odd integers is an odd integer.
5. e.g.,

n	n
$\times 2$	$2n$
$+ 20$	$2n + 20$
$\div 2$	$n + 10$
$- n$	10

6. Darlene, Andy, Candice, Bonnie
7. The proof is valid; all the steps are correct.

Chapter Review, page 61

1. e.g., The diagonals of parallelograms always bisect each other. The diagrams in the question support my conjecture.
2. e.g.,
   a) The difference between consecutive triangular numbers increases by 1: 2, 3, 4, … The next four triangular numbers are 15, 21, 28, and 36.
   b) Each of the products is double the first, second, third, and fourth triangular numbers, respectively.
   c) The nth triangular number could be determined using the formula $\frac{n(n + 1)}{2}$.
3. e.g.,
   a) The sum of the cubes of the first n natural numbers is equal to the square of the nth triangular number.
   b) The next equation will be equal to 15^2, or 225.
   c) The sum of the first n cubes will be equal to $\left(\frac{n(n + 1)}{2}\right)^2$.
4. e.g.,
   a) $37 \times 15 = 555$
   b) The conjecture is correct.
   c) The breakdown occurs at $37 \times 30 = 1110$.
5. e.g.,
   a) A counterexample is an example that invalidates a conjecture.
   b) Counterexamples can help refine a conjecture to make it valid.
6. Disagree. e.g., Rhombuses and parallelograms have opposite sides of equal length.
7. Disagree. e.g., $5 - 5 = 0$
8. Six is an even number; therefore, its square is also even.
9. e.g.,
   Let $2m + 1$ and $2n + 1$ represent any two odd integers.
   $(2m + 1)(2n + 1) = 2mn + 2m + 2n + 1$
   $(2m + 1)(2n + 1) = 2(mn + m + n) + 1$
   The first term has a factor of 2, making it an even number. Adding 1 makes the product odd.
10. **a)** The result is the birth month number followed by the birthday, e.g., 415.
   b)

m	m
$\times 5$	$5m$
$+ 7$	$5m + 7$
$\times 4$	$20m + 28$
$+ 13$	$20m + 41$
$\times 5$	$100m + 205$
$+ d$	$100m + 205 + d$
$- 205$	$100m + d$

   The birth month is multiplied by 100, leaving enough space for a two-digit birthday.
11. **a)** e.g., Twice the sum of the squares of two numbers is equal to the sum of the squared difference of the numbers and the squared sum of the numbers.
   b) Let n and m represent any two numbers.
   $2(n^2 + m^2) = 2n^2 + 2m^2$
   $2(n^2 + m^2) = n^2 + n^2 + m^2 + m^2 + 2mn - 2mn$
   $2(n^2 + m^2) = (n^2 - 2mn + m^2) + (n^2 + 2mn + m^2)$
   $2(n^2 + m^2) = (n - m)^2 + (n + m)^2$
   $2(n^2 + m^2) = a^2 + b^2$
   Let a represent $n - m$ and b represent $n + m$.
   A sum of two squares, doubled, is equal to the sum of two squares.
12. e.g., On the fourth line there is a division by zero, since $a = b$.
13. Julie did not multiply 10 by 5 in the third line.

n	Choose a number.
$n + 10$	Add 10.
$5n + 50$	Multiply the total by 5.
$5n$	Subtract 50.
5	Divide by the number you started with.

14. One of the women is both a mother and a daughter.
15.

Penny Pig	straw	small	Riverview
Peter Pig	sticks	large	Hillsdale
Patricia Pig	brick	medium	Pleasantville

16. e.g., Player X should choose the bottom left corner, then the top left corner, then the middle left or middle, depending on where Player X was blocked.
17. e.g.,
   a) yes
   b) There is no winning strategy in the game of 15. An experienced opponent will always succeed in blocking you.

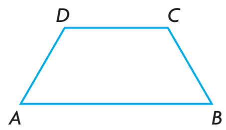
Chapter 2

Lesson 2.1, page 72

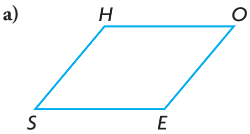
1. e.g.,
   a) Horizontal beams are parallel.
   Vertical supports are parallel.
   Diagonal struts are transversals.
   b) No. The bridge is shown in perspective. Parallel lines on the bridge will not be parallel when they are traced, so corresponding angles will not be equal in the tracing.
2. $\angle EGB = \angle GHD$, $\angle AGE = \angle CHG$, $\angle AGH = \angle CHF$, $\angle BGH = \angle DHF$, $\angle EGA = \angle HGB$, $\angle EGB = \angle HGA$, $\angle GHD = \angle FHC$, $\angle GHC = \angle FHD$, $\angle EGA = \angle FHD$, $\angle EGB = \angle FHC$, $\angle GHD = \angle HGA$, $\angle GHC = \angle BGH$
   Yes, the measures are supplementary.
3. e.g., Draw a line and a transversal, then measure one of the angles between them. Use a protractor to create an equal corresponding angle elsewhere on the same side of the transversal. Use that angle to draw the parallel line.
4. e.g., The top edge of the wood is the transversal for the lines that are drawn. Keeping the angle of the T-bevel the same makes parallel lines because corresponding angles are equal.
5. **a)** No. Corresponding angles are not equal.
   b) Yes. Corresponding angles are equal.
   c) Yes. Corresponding angles are equal.
   d) No. Corresponding angles are not equal.
6. Disagree. The lines are equidistant from each other. It is an optical illusion.

Lesson 2.2, page 78

1. KP, LQ, MR, and NS are all transversals for the parallel lines WX and YZ.
   $\angle WYD = 90°$; $\angle WYD$ and $\angle AWY$ are interior angles on the same side of KP.
   $\angle YDA = 115°$; $\angle YDA$ and $\angle WAL$ are corresponding angles.
   $\angle DEB = 80°$; $\angle DEB$ and $\angle EBC$ are alternate interior angles.
   $\angle EFS = 45°$; $\angle EFS$ and $\angle NCX$ are alternate exterior angles.
2. **a)** Yes. Corresponding angles are equal.
   b) No. Interior angles on the same side of the transversal are not supplementary.
   c) Yes. Alternate exterior angles are equal.
   d) Yes. Alternate exterior angles are equal.
3. e.g.,
   a) Alternate interior angles are equal.
   b) Corresponding angles are equal.
   c) Alternate exterior angles are equal.
   d) Vertically opposite angles are equal.
   e) $\angle b$ and $\angle k$ and $\angle m$ are all equal in measure; $\angle b$ and $\angle k$ are corresponding angles, $\angle k$ and $\angle m$ are corresponding angles.
   f) $\angle e$ and $\angle n$ and $\angle p$ are all equal in measure; $\angle e$ and $\angle n$ are corresponding angles, $\angle n$ and $\angle p$ are corresponding angles.
   g) $\angle n$ and $\angle p$ and $\angle d$ are all equal in measure; $\angle n$ and $\angle p$ are corresponding angles, $\angle p$ and $\angle d$ are alternate exterior angles.
   h) $\angle f$ and $\angle k$ are interior angles on the same side of a transversal.
4. **a)** $\angle x = 60°$, $\angle y = 60°$, $\angle w = 120°$
   b) $\angle a = 112°$, $\angle e = 112°$, $\angle b = 55°$, $\angle d = 55°$, $\angle f = 55°$, $\angle c = 68°$
   c) $\angle a = 48°$, $\angle b = 48°$, $\angle c = 48°$, $\angle d = 48°$, $\angle e = 132°$, $\angle f = 132°$, $\angle g = 132°$
5. e.g.,



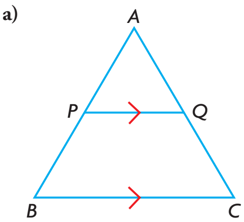
   I drew AB and used a protractor to create a 60° angle at A and at B. I drew BC and created a 120° angle at C, so that CD would be parallel to AB. Then I drew AD to intersect CD.
6. e.g.,
   a)



   b) $\angle S = 50°$
   $\angle H + \angle S = 180°$
   $\angle H = 130°$
   $\angle H + \angle O = 180°$
   $\angle O = 50°$
   $\angle S = \angle O$
7. **a)** e.g., The horizontal lines in the fabric are parallel and the diagonal lines are transversals. The diagonal lines falling to the right are parallel and the diagonal lines rising to the right are transversals.
   b) e.g., A pattern maker could ensure that lines in the pattern are parallel by making the corresponding, alternate exterior, or alternate interior angles equal, or by making the angles on the same side of a transversal supplementary.
8. **a)** The transitive property is true for parallel lines but not for perpendicular lines.
   b) If $AB \perp BC$ and $BC \perp CD$, then $AB \parallel CD$.
9. e.g., Theoretically, they could measure corresponding angles to see if they were equal.
10. e.g., errors: interior angles should be stated as supplementary, not equal. Since $\angle PQR + \angle QRS = 180°$, the statement that $QP \parallel RS$ is still valid.
11. e.g., The bottom edges of the windows are transversals for the vertical edges of the windows. The sloped roof also forms transversals for the vertical parts of the windows. The builders could ensure one window is vertical and then make all the corresponding angles equal so the rest of the windows are parallel.
12. e.g.,

$SR \parallel XO$	$\angle FOX$ and $\angle FRS$ are equal corresponding angles
$PQ \parallel XO$	$\angle FPQ$ and $\angle FXO$ are equal corresponding angles.
$PQ \parallel SR$	Transitive property

13. e.g.,
   a)



   b)

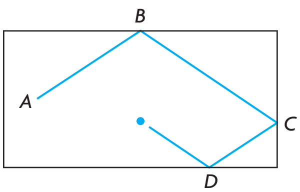
$\angle APQ = \angle ABC$	Corresponding angles
$\angle AQP = \angle ACB$	Corresponding angles
$\angle PAQ = \angle BAC$	Same angle
$\triangle APQ \sim \triangle ABC$	Corresponding angles in the two triangles are equal.

14. **a)** $\angle x = 120°$, $\angle y = 60°$, $\angle z = 60°$
b) e.g., Isosceles trapezoids have two pairs of congruent adjacent angles.

15. $\angle PTQ = 78°$, $\angle PQT = 48°$, $\angle RQT = 49°$, $\angle QTR = 102°$, $\angle SRT = 54°$, $\angle PTS = 102°$

16. $\angle ACD = \angle ACF + \angle FCD$
$\angle BAC = \angle ACF$
$\angle CDE = \angle FCD$
$\angle ACD = \angle BAC + \angle CDE$

17. **a)** Alternate straight paths will be parallel.
b)



c) $AB \parallel CD$, $BC \parallel DE$
d) Yes, the pattern will continue until the ball comes to rest.

18. e.g.,

$\angle PQR = \angle QRS$	Alternate interior angles
$\frac{1}{2}\angle PQR = \frac{1}{2}\angle QRS$	Equality
$\angle TRQ = \frac{1}{2}\angle PQR$	Angle bisector
$\angle RQU = \frac{1}{2}\angle QRS$	Angle bisector
$\angle TRQ = \angle RQU$	Transitive property
$QU \parallel RT$	Alternate interior angles are equal.

19. e.g.,
a) Disagree; it is enough to show that any one of the statements is true.
b) Yes. Other ways are $\angle MCD = \angle CDQ$, $\angle XCL = \angle CDQ$, $\angle LCD + \angle CDQ = 180°$, $\angle LCD = \angle QDY$, $\angle MCD = \angle RDY$, $\angle XCM = \angle QDY$, or $\angle XCL = \angle RDY$.

20. **a)** 8 **b)** 7

21. e.g.,
a) Measure the top angle of the rhombus at the left end of the bottom row; it will be the same size as the angle at the peak.
b) Opposite sides of a rhombus are parallel, so the top right sides of all the rhombuses form parallel lines. The top right side of the peak rhombus and the top right side of the bottom left rhombus are parallel. The left edge of the pyramid is a transversal, so the angle at the peak and the top angle of the bottom left rhombus are equal corresponding angles.

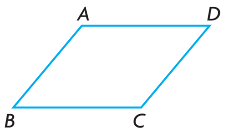
Mid-Chapter Review, page 85

1. **a)** Yes. Alternate interior angles are equal.
b) No. Interior angles on the same side of the transversal are not supplementary.
c) Yes. Alternate exterior angles are equal.
d) Yes. Vertically opposite angles are equal and interior angles on the same side of the transversal are supplementary.

2. Quadrilateral *PQRS* is a parallelogram because interior angles on the same side of the transversal are supplementary.

3. e.g., The red lines are parallel since any of the black lines can be used as a transversal to prove that corresponding angles are equal.

4. e.g.,



I drew $\angle ABC$. I measured it and drew $\angle BCD$ supplementary to it. Then I measured *AB*, made *CD* the same length, and connected *A* to *D*.

5. **a)** $\angle FEB = 69°$, $\angle EBD = 69°$, $\angle FBE = 36°$, $\angle ABF = 75°$, $\angle CBD = 75°$, $\angle BDE = 75°$
b) Yes. e.g., $\angle FEB$ and $\angle EBD$ are equal alternate interior angles.

6. e.g.,
a)

$AC \parallel ED$	$\angle ABE$ and $\angle BED$ are equal alternate interior angles.

b)

$\angle BED = 55°$ and $\angle BFG = \angle BED$, therefore $\angle BFG = 55°$	$\angle BFG$ and $\angle BED$ are corresponding angles in $\triangle BFG \sim \triangle BED$.
$FG \parallel ED$	$\angle BFG$ and $\angle BED$ are equal corresponding angles for *FG* and *ED*.

c)

$AC \parallel FG$	$\angle ABF$ and $\angle BFG$ are equal alternate interior angles.

7. e.g., In each row of parking spots, the lines separating each spot are parallel. The line down the centre is the transversal to the two sets of parallel lines.

8. e.g., Yes, the sides are parallel. The interior angles are supplementary and so the lines are always the same distance apart.

Lesson 2.3, page 90

1. No. It only proves the sum is 180° in that one triangle.

2. Disagree. The sum of the three interior angles in a triangle is 180°.

3. **a)** $\angle YXZ = 79°$, $\angle Z = 37°$
b) $\angle DCE = 46°$, $\angle A = 85°$

4. $\angle R = \frac{1}{2}(180° - n°)$

5. e.g.,

$\angle CDB = 60°$	$\triangle BCD$ is equilateral.
$\angle CDB + \angle BDA = 180°$ $\angle BDA = 120°$	$\angle CDB$ and $\angle BDA$ are supplementary.
$\angle A = \frac{1}{2}(180° - 120°)$ $\angle A = 30°$	Since $\triangle BDA$ is an isosceles triangle, $\angle A$ and $\angle B$ are equal.

6. 120°

7. e.g.,

$\angle ASY = 53°$	Sum of angles in triangle is 180°.
$\angle SAD = 127°$	Given
$\angle ASY + \angle SAD = 180°$	Property of equality
$SY \parallel AD$	Interior angles on the same side of the transversal are supplementary.

Answers

8. e.g.,

a) The sum of $\angle a$, $\angle c$, and $\angle e$ is 360°.

b) Yes. $\angle b = \angle a$, $\angle d = \angle c$, $\angle f = \angle e$

c)

$\angle x + \angle a = 180°$ $\angle a = 180° - \angle x$	$\angle x$ and $\angle a$ are supplementary.
$\angle y + \angle c = 180°$ $\angle c = 180° - \angle y$	$\angle y$ and $\angle c$ are supplementary.
$\angle z + \angle e = 180°$ $\angle e = 180° - \angle z$	$\angle z$ and $\angle e$ are supplementary.
$\angle a + \angle c + \angle e$ $= (180° - \angle x) +$ $(180° - \angle y) + (180° - \angle z)$ $\angle a + \angle c + \angle e$ $= 540° - (\angle x + \angle y + \angle z)$	I substituted the expressions for $\angle a$, $\angle c$, and $\angle e$.
$\angle a + \angle c + \angle e = 540° - 180°$ $\angle a + \angle c + \angle e = 360°$	$\angle x$, $\angle y$, and $\angle z$ are the angles of a triangle so their sum is 180°.

9. e.g.,

a) $\angle D \neq \angle C$

b)

$\angle DKU = \angle KUC$ $\angle DKU = 35°$	$\angle DKU$ and $\angle KUC$ are alternate interior angles.
$\angle DUK = 180° - (100° + 35°)$ $\angle DUK = 45°$	The sum of the angles of $\triangle DUK$ is 180°.
$\angle UKC = 45°$	$\angle DUK$ and $\angle UKC$ are alternate interior angles.
$\angle UCK = 100°$	Opposite angles in a parallelogram are equal.

10. e.g.,

$MA \parallel HT$	$\angle MTH$ and $\angle AMT$ are equal alternate interior angles.
$MH \parallel AT$	$\angle MHT = 70°$ and $\angle HTA = 45° + 65°$ are supplementary interior angles on the same side of transversal HT.

11. $\angle a = 30°$, $\angle b = 150°$, $\angle c = 85°$, $\angle d = 65°$

12. e.g.,

a) Disagree. $\angle FGH$ and $\angle IHJ$ are not corresponding angles, alternate interior angles, or alternate exterior angles.

b)

$\angle GFH = 180° - (55° + 75°)$ $\angle GFH = 50°$	The sum of the angles of $\triangle FGH$ is 180°.
$FG \parallel HI$	$\angle GFH$ and $\angle IHJ$ are equal corresponding angles.

13. $\angle J = 110°$, $\angle M = 110°$, $\angle JKO = 40°$, $\angle NOK = 40°$, $\angle KLN = 40°$, $\angle LNM = 40°$, $\angle MLN = 30°$, $\angle JOK = 30°$, $\angle LNO = 140°$, $\angle KLM = 70°$, $\angle JON = 70°$

14. $\angle UNF = 31°$, $\angle NFU = 65°$, $\angle FUN = 84°$

15. **a)** $\angle AXZ = 145°$, $\angle XYC = 85°$, $\angle EZY = 130°$

b) 360°

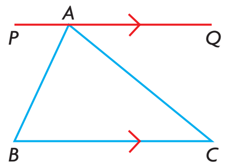
16. e.g.,

MO and NO are angle bisectors.	Given
$\angle LNP$ is an exterior angle for $\triangle LMN$.	
$\angle L + 2a = 2b$ $\angle L = 2b - 2a$ $\angle L = 2(b - a)$	An exterior angle is equal to the sum of the non-adjacent interior angles.
$\angle ONP$ is an exterior angle for $\triangle MNO$.	
$\angle O + a = b$ $\angle O = b - a$	An exterior angle is equal to the sum of the non-adjacent interior angles.
$\angle L = 2(b - a)$ $\angle L = 2\angle O$	Substitution

17. e.g., Drawing a parallel line through one of the vertices and parallel to one of the sides creates three angles whose sum is 180°. The two outside angles are equal to the alternate interior angles in the triangle. The middle angle is the third angle in the triangle. Therefore, the three angles in the triangle add up to 180°.

$\angle PAB = \angle ABC$

$\angle QAC = \angle ACB$

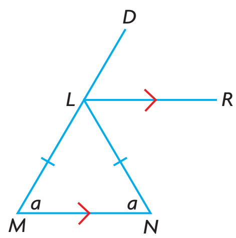


18. e.g.,

$\angle DAB + \angle ABD + \angle BDA = 180°$	The sum of the angles of $\triangle ABD$ is 180°.
$2\angle x + (90° + \angle y) + \angle y = 180°$ $2\angle x + 2\angle y = 90°$ $\angle x + \angle y = 45°$	
$\angle AEB = \angle x + \angle y$	$\angle AEB$ is an exterior angle for $\triangle AED$, so it is equal to the sum of the non-adjacent interior angles.
$\angle AEB = 45°$	Substitute $\angle x + \angle y = 45°$.

19. e.g.,

$\angle DLR = \angle LMN$	Corresponding angles
$\angle RLN = \angle LNM$	Alternate interior angles
$\angle LMN = \angle LNM$	Isosceles triangle
$\angle DLR = \angle RLN$	Transitive property



Lesson 2.4, page 99

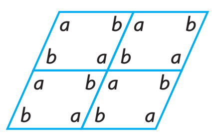
1. **a)** 1800° **b)** 150°

2. 3240°

3. 19

4. e.g., The interior angles of a hexagon equal 120°. Three hexagons will fit together since the sum is 360°.

5. Yes. e.g., You can align parallel sides to create a tiling pattern; the angles that meet are the four angles of the parallelogram, so their sum is 360°.



6. about 147°

7. e.g.,

a) $\dfrac{180°(n-2)}{n} = 140°$

$$180°\,(n-2) = 140°n$$
$$180°n - 360° = 140°n$$
$$40°n = 360°$$
$$n = 9$$

b) There are 9 exterior angles that measure $180° - 140° = 40°$; $9(40°) = 360°$.

8. **a)** 45° **c)** 1080°
b) 135° **d)** 1080°

9. **a)** Agree
b) e.g., Opposite sides are parallel in a regular polygon that has an even number of sides.

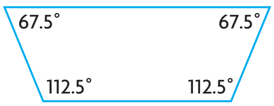
10. **a)** 36° **b)** isosceles triangle

11. The numerator of the formula for $S(10)$ should be $180°(10-2)$; $S(10) = 144°$.

12. **a)** e.g., A single line drawn anywhere through the polygon. For convex polygons, it intersects two sides only. For non-convex polygons, it can intersect in more than two sides.
b) If any diagonal is exterior to the polygon, the polygon is non-convex.

13. **a)**

b)



14. 110°, 120°, 90°, 110°, 110°

15. 360°

16. **a)** $\angle a = 60°$, $\angle b = 60°$, $\angle d = 60°$, $\angle c = 120°$
b) $\angle a = 140°$, $\angle b = 20°$, $\angle c = 60°$, $\angle d = 60°$

17. 720°

18. e.g.,

$\triangle EOD \cong \triangle DOC$	$EO = DO$ and $DO = CO$ are given, and $ED = DC$ because the polygon is regular.
$\angle ODE = \angle ODC$ and $\angle ODE = \angle OED$	$\triangle EOD$ and $\triangle DOC$ are congruent and isosceles.
$\angle ODE + \angle ODC = 108°$	The interior angles of a regular pentagon are 108°.
$2\angle ODE = 108°$ $\angle ODE = 54°$ $\angle OED = 54°$	$\angle ODE = \angle ODC$ and $\angle ODE = \angle OED$
$\angle EAD = \angle EDA$	$\triangle ADE$ is isosceles because the polygon is regular.
$180° = \angle DEA + \angle EAD + \angle ADE$ $180° = 108° + 2\angle ADE$ $180° - 108° = 2\angle ADE$ $36° = \angle ADE$	
$180° = \angle FED + \angle EDF + \angle EFD$ $180° = 54° + 36° + \angle EFD$ $180° - 54° - 36° = \angle EFD$ $90° = \angle EFD$	$\angle EDF = \angle ADE$ and $\angle FED = \angle OED$

19. e.g., If a polygon is divided into triangles by joining one vertex to each of the other vertices, there are always two fewer triangles than the original number of sides. Every triangle has an angle sum of 180°.

20. Yes, e.g., A tiling pattern can be created by putting four 90° angles together or three 120° angles together.

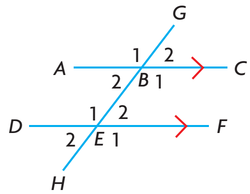
21. regular dodecagon

Chapter Self-Test, page 104

1. **a)** $a = 70°$, $b = 75°$, $c = 75°$
b) $a = 20°$, $b = 80°$, $c = 100°$

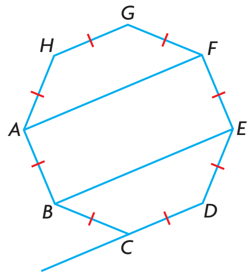
2. **a)** $x = 19°$ **b)** $x = 26°$

3. **a)** and **c)** e.g.,



4. regular hexagons: six 120° angles; small triangles: three 60° angles; large triangles: one 120° angle and two 30° angles.

5. **a)**



b) 45°

c)

Extend BC to form exterior angles $\angle ABI$ and $\angle DCJ$.	
$\angle ABI = 45°$ $\angle DCJ = 45°$	Exterior angle of regular octagon
$BE \parallel CD$	Alternate exterior angles are equal.
$\angle CBE = 45°$	Alternate interior angles
$\angle ABE = 90°$	Supplementary angles
Similarly, by extending AH and following the process above, $\angle FAB = 90°$.	
$\angle ABE + \angle FAB = 180°$	
$AF \parallel BE$	Interior angles on the same side of the transversal are supplementary.

6. 720°

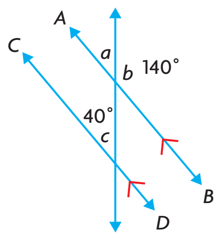
Chapter Review, page 106

1. e.g., The side bars coming up to the handle are parallel and the handle is a transversal.

2. **a)** $\angle a$, $\angle e$; $\angle b$, $\angle g$; $\angle c$, $\angle f$; $\angle d$, $\angle h$
b) No. e.g., The lines are not parallel, so corresponding pairs cannot be equal.
c) 8; e.g., $\angle a$, $\angle b$
d) Yes; $\angle a$, $\angle d$; $\angle b$, $\angle c$; $\angle e$, $\angle h$; $\angle f$, $\angle g$

3. $\angle a = 35°$, $\angle b = 145°$

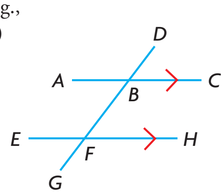
4.



$\angle a + \angle b = 180°$	$\angle a$ and $\angle b$ form a straight angle.
$\angle a = 40°$	Substitution and subtraction
$\angle c = 40°$	Given
$\angle a = \angle c$	Corresponding angles are equal.
$AB \parallel CD$	

5. **a)** $a = 104°, b = 76°, c = 76°$
b) $a = 36°, b = 108°, c = 108°$

6. e.g.,
a)



b) Measure $\angle ABF$ and $\angle BFH$. Measure $\angle DBA$ and $\angle BFE$. Both pairs should be equal.

7. e.g.,

$\angle QRS = \angle RST$	Alternate interior angles
$\angle QRS = \angle TRS$	Given
$\angle RST = \angle TRS$	Transitive property
$TS = TR$	Isosceles triangle

8. **a)** $x = 40°, y = 95°, z = 45°$
b) $x = 68°, y = 112°, z = 40°$

9. e.g.,

$\angle OPL = \angle POL$ $\angle OQN = \angle NOQ$	$\triangle OPL$ and $\triangle NOQ$ are isosceles.
$\angle PLO = 180° - (\angle POL + \angle OPL)$ $\angle QNO = 180° - (\angle NOQ + \angle OQN)$	The sum of the angles in each triangle is 180°.
$\angle PLO = 180° - 2\angle POL$ $\angle QNO = 180° - 2\angle NOQ$	Substitute $\angle OPL = \angle POL$ and $\angle OQN = \angle NOQ$.
$\angle PLO + \angle QNO = 180° - 90°$ $\angle PLO + \angle QNO = 90°$	$\angle PLO$ and $\angle QNO$ are the two acute angles in the right triangle LMN.
$(180° - 2\angle POL) + (180° - 2\angle NOQ) = 90°$	Substitute the expressions for $\angle PLO$ and $\angle QNO$.
$\angle POL + \angle NOQ = 135°$	Isolate $\angle POL + \angle NOQ$ in the equation.
$\angle POQ = 45°$	$\angle POQ$, $\angle POL$, and $\angle NOQ$ are supplementary because they form a straight line.

10. **a)** 2340°
b) e.g., The sum of the 15 exterior angles is 360°, so each exterior angle is $360° \div 15 = 24°$.

11. e.g.,

$\angle ABC = 108°$, $\angle BCD = 108°$, $\angle CDE = 108°$	The angles in a regular pentagon are 108°.
$\angle BCA + \angle BAC = 180° - 108°$	The sum of the angles of $\triangle ABC$ is 180°.
$2\angle BCA = 72°$ $\angle BCA = 36$	$\triangle ABC$ is isosceles with $\angle BCA = \angle BAC$.
$\angle ACD = \angle BCD - \angle BCA$ $\angle ACD = 108° - 36°$ $\angle ACD = 72°$	$\angle BCA + \angle ACD = \angle BCD$
$AC \parallel ED$	$\angle ACD = 72°$ and $\angle CDE = 108°$ are supplementary interior angles on the same side of the transversal CD.

Cumulative Review, Chapters 1–2, page 110

1. e.g.,
a) A conjecture is a testable expression that is based on available evidence but is not yet proven.
b) Inductive reasoning involves looking at examples, and by finding patterns and observing properties, a conjecture may be made.
c) The first few examples may have the same property, but that does not mean that all other cases will have the same property. e.g., Conjecture: The difference of consecutive perfect cubes is always a prime number.
$2^3 - 1^3 = 7$ $\quad$ $5^3 - 4^3 = 61$
$3^3 - 2^3 = 19$ $\quad$ $6^3 - 5^3 = 91$,
$4^3 - 3^3 = 37$ $\quad$ 91 is not a prime number.

2. Yes, her conjecture is reasonable.

3. One. e.g., Conjecture: All prime numbers are odd numbers. 2 is a prime number but is not odd.

4. Disagree.

5. **a)** Conjecture: The sum of two odd numbers is always an even number.
b) e.g., Let $2n + 1$ and $2k + 1$ represent any two odd numbers.
$(2n + 1) + (2k + 1) = 2n + 2k + 2 = 2(n + k + 1)$
$2(n + k + 1)$ is an even number.

6. e.g.,

Instruction	Result
Choose a number.	x
Double it.	$2x$
Add 9.	$2x + 9$
Add the number you started with.	$2x + 9 + x = 3x + 9$
Divide by 3.	$\frac{(3x + 9)}{3} = x + 3$
Add 5.	$x + 3 + 5 = x + 8$
Subtract the number you started with.	$(x + 8) - x = 8$

7. **a)** The number of circles in the nth figure is $1 + 5(n - 1) = 5n - 4$; there are 71 circles in the 15th figure.
b) Inductive. A pattern in the first few cases was used to come up with a formula for the general case.

8. Let $ab0$ represent the three digit number. Then, $ab0 = 100a + 10b = 10(10a + b)$, which is divisible by 10.

9. e.g., Turn one of the switches on for a short period of time and then turn it off. Turn on another of the switches and leave it on. Enter the room. Check which of the two light bulbs that is off is still warm. This light belongs to the switch that was turned on and then off. The light bulb that is on belongs to the switch that was left on. The last light bulb belongs to the last switch.

10. **a)** $\angle a = 75°$, $\angle b = 105°$, $\angle c = 105°$, $\angle d = 105°$
b) $\angle a = 50°$, $\angle f = 50°$, $\angle b = 55°$, $\angle e = 55°$, $\angle c = 75°$, $\angle d = 75°$
c) $\angle x = 50°$, $\angle y = 60°$
d) $\angle a \doteq 128.6°$, $\angle b \doteq 51.4°$

11. e.g., equal alternate interior angles, $\angle AEF = \angle DFE$.

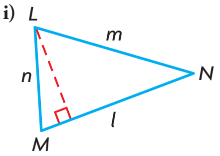
12. **a)** 540°
b) 108°
c) 360°

13. e.g. Any convex polygon can be divided into triangles by joining one vertex to the other vertices. There will be two fewer triangles than the number of sides. Each triangle has an angle sum of 180°. Therefore, the formula $S(n) = 180°(n - 2)$ will give the angle sum of any convex polygon.

Chapter 3

Lesson 3.1, page 117

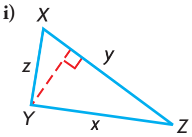
1. **a)** **i)**



ii) $h = m \sin N$, $h = n \sin M$

$$\frac{m}{\sin M} = \frac{n}{\sin N}$$

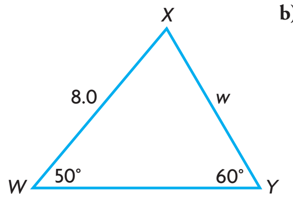
b) **i)**



ii) $h = z \sin X$, $h = x \sin Z$

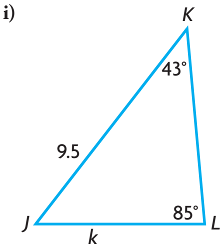
$$\frac{x}{\sin X} = \frac{z}{\sin Z}$$

2. **a)** **i)**



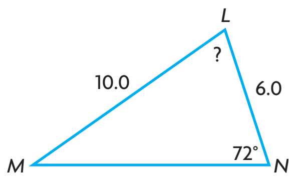
ii) $w = 7.1$

b) **i)**



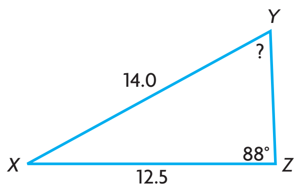
ii) $k = 6.5$

c) **i)**



ii) $\angle M = 34.8°$

d) **i)**

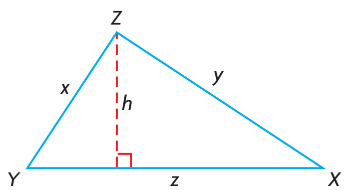


ii) $\angle Y = 63.2°$

3. Agree. $\sin X = \frac{h}{y}$ $\quad \sin Y = \frac{h}{x}$

$h = y \sin X \quad h = x \sin Y$

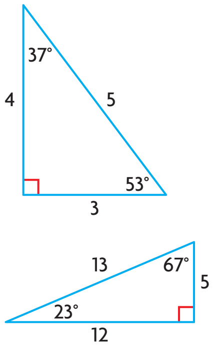
$\therefore y \sin X = x \sin Y$



4. e.g., You need two sides and the angle opposite one of the sides or two angles and any side.

Answers

5. e.g., Yes, the ratios are equivalent.



$\frac{3}{\sin 37°} = 5$

$\frac{4}{\sin 53°} = 5$

$\frac{5}{\sin 90°} = 5$

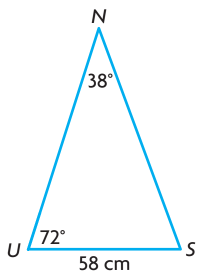
$\frac{5}{\sin 23°} = 13$

$\frac{12}{\sin 67°} = 13$

$\frac{13}{\sin 90°} = 13$

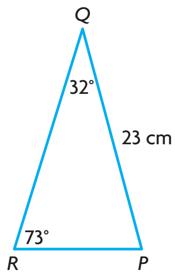
Lesson 3.2, page 124

1. $\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$
2. **a)** $b = 37.9$ cm **b)** $\theta = 61°$
3. **a)** $d = 21.0$ cm **d)** $\theta = 64°$
   b) $a = 26.1$ cm, $b = 35.2$ cm **e)** $\theta = 45°, \alpha = 85°$
   c) $y = 6.5$ cm **f)** $\theta = 25°, \alpha = 75°, j = 6.6$ m
4. **a)** e.g., The lake's length is opposite the largest angle of the triangle and must also be the longest side. A length of 36 km would not make it the longest side.
   b) 48.7 km
5. 32 ft 5 in.
6. **a)**



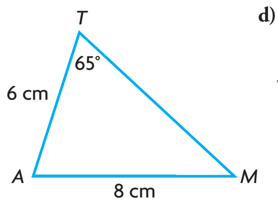
$u = 90$ cm

b)



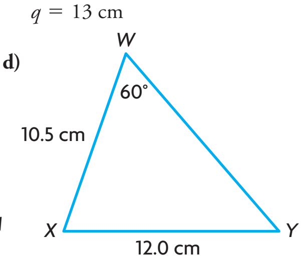
$q = 13$ cm

c)



$\angle M = 43°$

d)



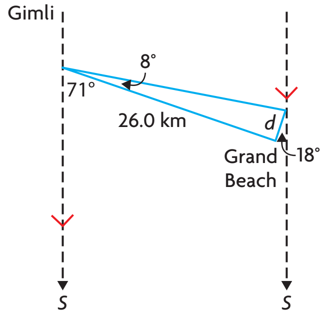
$\angle Y = 49°$

7. $a = 41.9$ m, $t = 44.9$ m, $\angle A = 67°$
8. **a) i)** $\sin 36.9 = \frac{n}{10}$, $n = 6.0$ cm

   ii) $\frac{10}{\sin 90°} = \frac{n}{\sin 36.9°}$, $n = 6.0$ cm

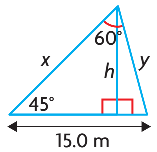
   b) e.g., Since $\sin 90° = 1$, you can rearrange the sine law formula to give the expression for the sine ratio.

9. **a)**



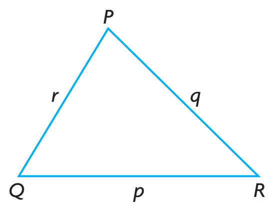
b) 3.6 km

10. **a)** e.g.,



b) The wires are 12.2 m and 16.7 m long, and the pole is 11.8 m high.

11. e.g., Use the Pythagorean theorem to determine the value of q, then use a primary trigonometric ratio to determine $\angle P$. $\sin P = \frac{8}{q} = \frac{8}{10}$
    Use the Pythagorean theorem to determine the value of q, then use the sine law to determine $\angle P$. $\frac{8}{\sin P} = \frac{10}{\sin 90°}$
12. 11.4 km
13. 24.8 m
14. e.g.,

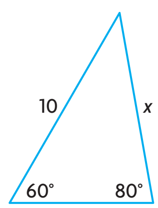


a) $\angle P, \angle R, q$
b) $\angle P, q, r$

15. Agree. Jim needs to know an angle and its opposite side.
16. e.g., You can determine $\angle R$ since the sum of the three angles of a triangle is 180°; you can use the sine law to determine q and r.
17. 19.7 square units
18. 10.2 cm
19. e.g.,
    a) $\frac{a}{b}$ **b)** $\frac{\sin A}{\sin C}$ **c)** 1

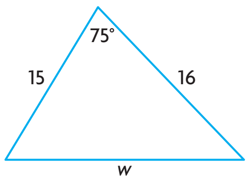
Mid-Chapter Review, page 129

1. $\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$ or $\frac{\sin X}{x} = \frac{\sin Y}{y} = \frac{\sin Z}{z}$
2. **a)** e.g., **b)** $x = 8.8$

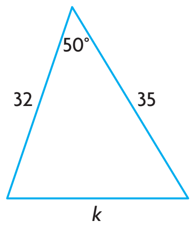


3. e.g., Disagree; you can't rearrange Nazir's expression so that f and $\sin F$ are in one ratio and d and $\sin D$ are in the other.
4. a) $x = 5.9$ cm, $\theta = 42.9°$
   b) $x = 10.6$ cm, $y = 9.7$ cm, $\theta = 62.0°$
5. a) $\angle C = 60°$, $b = 12.2$ cm, $c = 13.8$ cm
   b) $\angle L = 85°$, $l = 32.9$ cm, $m = 32.7$ cm
6. e.g., The value of either $\angle X$ or $\angle Z$ is needed to solve the triangle.
7. a) The tower at B is closer. e.g., The distance from tower B to the fire is length a, which is across from the smaller angle.
   b) 3.1 km
8. 631 m
9. a) 84.2 cm b) 82.3 cm

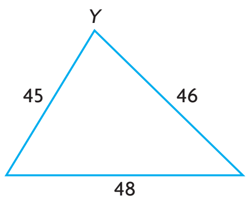
Lesson 3.3, page 136

1. a) No b) Yes
2. 13 cm
3. $\angle P = 72°$
4. a) 6.9 cm b) 14.7 cm
5. a) 34° b) 74°
6. e.g.,
   a)


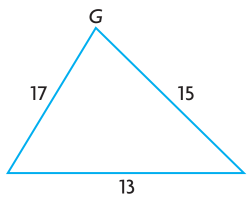
   $w = 18.9$

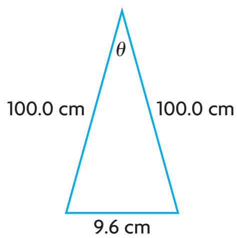
   b)


   $k = 28.4$

   c)


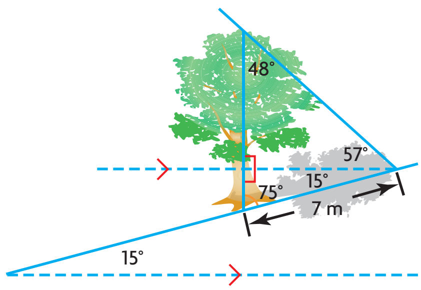
   $\angle Y = 63.7°$

   d)


   $\angle G = 47.4°$
7. a) $f = 6.3$ cm, $\angle D = 45.9°$, $\angle E = 69.1°$
   b) $r = 10.1$ m, $\angle P = 38.6°$, $\angle Q = 61.4°$
   c) $\angle L = 86.6°$, $\angle M = 56.6°$, $\angle N = 36.8°$
   d) $\angle X = 75.2°$, $\angle Y = 48.0°$, $\angle Z = 56.8°$
8. a)


   b) 5.5°
9. 53.0 cm
10. e.g., You can use the cosine law; the 70° angle is one of the acute angles across from the shorter diagonal. It is contained between an 8 cm side and a 15 cm side.
11. a) i) about 17 cm
    ii) about 17 cm
    b) e.g., The hour and minute hands are the same distance apart at 2:00 and 10:00, and the triangles formed are congruent.
12. No. e.g., When you put the side lengths into the cosine law expression, you do not get $-1 \leq \cos\theta \leq 1$.
13. 34.4 km
14. e.g., Kathryn wants to determine the length of a pond. From where she is standing, one end of the pond is 35 m away. If she turns 35° to the left, the distance to the other end of the pond is 30 m. How long is the pond? Use the cosine law to determine the unknown side length.
15. 423 cm^2
16. area $\doteq 8.2$ cm^2; perimeter $\doteq 10.9$ cm
17. e.g., The vertex angle at the handle of the knife is about 110°. Each of the sides of the knife is about 8.5 cm in length.

Lesson 3.4, page 147

1. a) sine law
   b) tangent ratio or sine law
   c) cosine law
2. a) part a: $\theta = 83.9°$, part b: $c = 1.9$ cm, part c: $\theta = 39.6°$
   b) e.g., Using a trigonometric ratio is more efficient because you have fewer calculations to do.
3. a) Using the cosine law. b) 2.5 km
4. 29' 2", 31' 3"
5. a) 43.2 m b) about 13.3 m
6. a) e.g., Use the properties of parallel lines to determine the angle from the shadow up to the horizontal. Subtract that angle from 57° to determine the angle from the horizontal up to the sun. Both of these are angles of right triangles with one side along the tree. Subtract each angle from 90° to determine the third angle in each right triangle. Use the sine law to determine the height of the tree.


   b) 8 m
7. 241.2 m
8. 293.9 m
9. a) 11.1 m b) 18.8 m
10. a) e.g., Connect the centre to the vertices to create congruent isosceles triangles and determine the angles at the centre. In one triangle, use the cosine law to determine the pentagon side length and multiply that answer by five.
    b) 58.8 cm
11. a) 879.3 m b) about 40 s
12. a) 157.0 km
    b) The airplane that is 100 km away will arrive first.
13. 85°, 95°, 85°, 95°
14. 520.2 m; e.g.,
    Step 1 – Determine $\angle BDC$ in $\triangle BDC$.
    Step 2 – Use the sine law to determine CD.
    Step 3 – In $\triangle ADC$, use the tangent ratio to determine h.
15. e.g., Starr and David leave school from the same spot. Starr walks N65°E at 3 km/h while David walks S30°E at 4 km/h. How far apart are they after 20 min? The problem can be solved using the cosine law.

16. **a)** 63° **b)** 52°
17. 50.0 cm^2

Chapter Self-Test, page 152

1. **a)** $\theta = 42.6°$ **b)** c = 2.4 cm
2. $\angle R = 52°$, $p = 25.0$ cm, $q = 18.9$ cm
3. 117.0 km
4. 11.6 cm
5. 130.5 m
6. 28.3 m^2
7. e.g., If the angle is the contained angle, then use the cosine law. If it is one of the other angles, use the sine law to determine the other non-contained angle, calculate the contained angle by subtracting the two angles you know from 180°, then use the cosine law.
8. e.g., When two angles and a side are given, the sine law must be used to determine side lengths. When two sides and the contained angle are given, the cosine law must be used to determine the third side.

Chapter Review, page 154

1. No. e.g., $\angle C = 90°$, so this will be a right triangle.
2. Part d) is incorrect.
3. **a)** $x = 23.7$ m **b)** $\theta = 61.9°$
4. $\angle C = 55°$, $a = 9.4$ cm, $b = 7.5$ cm
5. 295.4 m
6. Part a) is not a form of the cosine law.
7. **a)** $x = 7.6$ m **b)** $\theta = 68.2°$
8. $a = 12.2$ cm, $\angle B = 44.3°$, $\angle C = 77.7°$
9. 58°
10. 11.1 m
11. 584 km
12. 5.5 km, N34.9°W

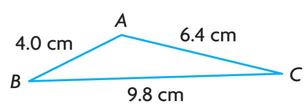
Chapter 4

Lesson 4.1, page 163

1. **a)** not valid **c)** not valid **e)** not valid
b) valid **d)** valid **f)** valid
2. **a)** 0.2588, 165° **c)** 0.7002, 145°
b) 0.4695, 118° **d)** 0.1736, 10°
3. **a)** 40°, 140° **c)** 72°, 108°
b) 19°, 161° **d)** 18°, 162°
4. **a)** $\sin B = \sin C$, $\sin B = \sin L$, $\sin C = \sin L$, $\sin D = \sin K$, $\sin E = \sin F$, $\sin H = \sin M$, $\sin H = \sin N$, $\sin H = \sin O$, $\sin I = \sin J$, $\sin M = \sin N$, $\sin M = \sin O$, $\sin N = \sin O$
b) The cosine and tangent ratios for $\angle D$ and $\angle K$, for $\angle M$ and $\angle H$, for $\angle N$ and $\angle H$, and for $\angle O$ and $\angle H$ are opposites. The other angles have equal cosine and tangent ratios.

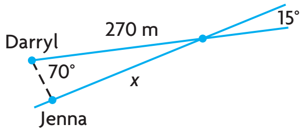
Lesson 4.2, page 170

1. **a)** sin 32° and sin 100° should be reversed.
b) On the left side of the equation, change 12 for x; on the right side of the equation, change x for 12.
2. **a)** sine law **d)** sine law
b) cosine law **e)** neither
c) cosine law
3. **a)** 8.4 cm **b)** 16.0 cm **c)** 3.1 cm
4. **a)** 35° **b)** 108° **c)** 17°
5. **a)** $m = 15.0$ cm, $\angle L = 46°$, $\angle N = 29°$
b) $t = 13.9$ cm, $r = 15.7$ cm, $\angle R = 32°$
c) $\angle A = 98°$, $\angle B = 30°$, $\angle C = 52°$
d) $y = 8.1$ cm, $z = 12.9$ cm, $\angle X = 124°$
6. **a)** e.g., about 135°

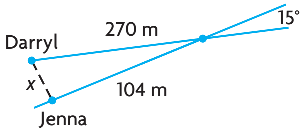


b) 139.8°
c) e.g., The estimate was reasonable. It could be improved by rounding the side lengths and using the cosine law.
7. Wei-Ting made a mistake from line 3 to line 4. The domain of inverse cosine is -1 to 1; 100 is outside the domain. $\theta = 130.5°$
8. $\angle Q = 23.8°$, $\angle R = 125.7°$, $\angle S = 30.5°$
9. 150 yd
10. within 8.1°
11. $f = 65.3$ m, $\angle D = 22.5°$, $\angle F = 21.5°$
12. 15.1 m
13. 257.0 m
14. 4139 m
15. e.g., Use the sine law: write an equation with sin R over r on the left side, and sin Q over q on the right side. Solve for sin Q. Determine Q by using $\sin^{-1} Q$ and subtracting from 180°.

16. **a)** e.g., Darryl sees his friend Jenna standing on the other road. He estimates the angle between the road he is on and his line of sight to Jenna to be 70°. How far is Jenna from the intersection? 255 m



b) e.g., Darryl sees his friend Jenna standing on the other road. If Jenna is 104 m from the intersection, how far apart are Jenna and Darryl? 172 m

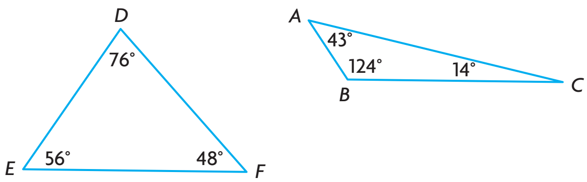


17. 35 cm

Mid-Chapter Review, page 175

1. **a)** 0.9659, 105°
b) −0.1736, 80°
c) 0.6249, 148°
d) 0.1392, 8°
e) 0.7826, 141.5°
f) −1.5818, 57.7°

2. e.g., sin 56° = sin 124°



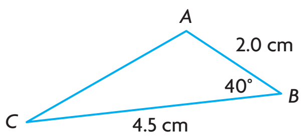
3. **a)** 21°, 159°
b) 139°
c) 68°
d) 30°, 150°
e) 78°
f) 45°

4. **a)** $\theta = 45.4°$ **b)** $y = 4.9$ km **c)** $x = 1.9$ cm

5. 14°

6. **a)** **i)** $\theta = 126.9°$ **ii)** $\alpha = 140.0°$
b) e.g., The diagram is needed for part i), since the acute angle, 53.1°, and the obtuse angle, 126.9°, have the same sine ratio. The diagram shows the angle is obtuse.

7. e.g.,
a) I cannot use the sine law because the measured angle is not across from a measured side.



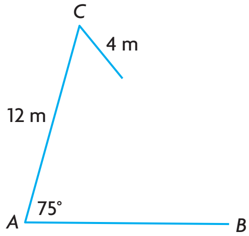
b) $\angle A = 117°$

8. 156°

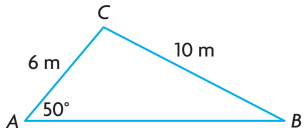
9. 147 km

Lesson 4.3, page 183

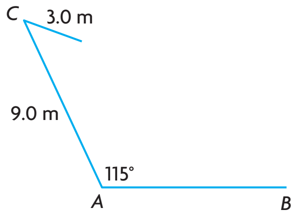
1. **a)** zero



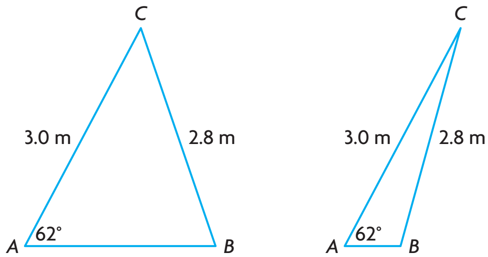
b) one



c) zero



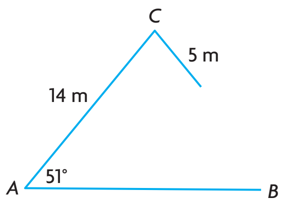
d) two



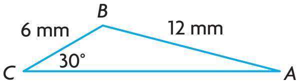
2. **a)** *SSA*
b) not *SSA*
c) *SSA*
d) not *SSA*
e) *SSA*
f) *SSA*

3. **a)** $h = 7.9$ cm, one
b) $h = 7.9$ cm or $h = 8.9$ cm, one
c) $h = 3.2$ cm, two
d) $h = 9.3$ cm or $h = 18.4$ cm, one
e) $h = 3.9$ cm, two
f) $h = 0.5$ cm, one

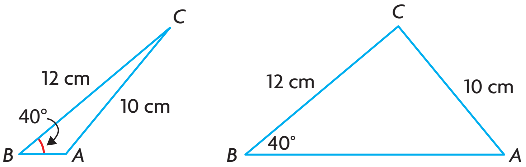
4. **a)** *SSA*, zero



b) *SSA*, one



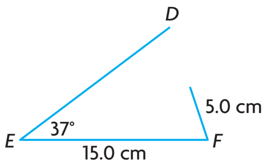
c) *SSA*, two



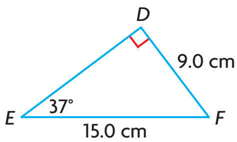
d) not *SSA*

5. **a)** 9.0 cm

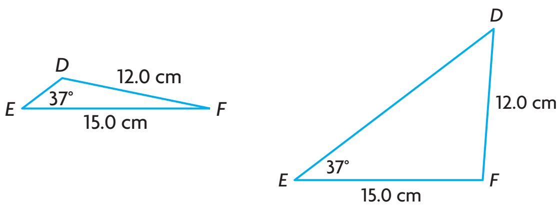
b) e.g., $FD = 5.0$ cm, zero triangles



$FD = 9.0$ cm, one triangle



$FD = 12.0$ cm, two triangles



6. **a)** 515 m

b) e.g., The 35° angle is opposite the side that is 430 m long.

7. **a)** 7.7 km

b) e.g., No. From the buoy, the canoe could have travelled 3 km toward the shore (acute angle) or away from the shore (obtuse angle).

8. **a)** 109.3°

b) e.g., Yes, there is only one possible answer. The 4.2 cm side must be opposite the obtuse angle. If it is not, then the triangle is an acute triangle.

9. 19.5 m and 8.2 m

e.g., 19.5 m, because there would be more vertical support

10. 80 m or 344 m

11. 3.1 km or 0.2 km

12. Carol on same side as 35° only. Distance to 66° is 11 m:

a) 9 m **b)** 11 m **c)** 5 m

Carol on same side as 35° only. Distance to 35° is 11 m:

a) 19 m **b)** 24 m **c)** 24 m

All on same side. Distance to 35° is 11 m:

a) 19 m **b)** 24 m **c)** 7 m

All on same side. Distance to 66° is 11 m:

a) 28 m **b)** 36 m **c)** 16 m

Carol on same side as neither. Distance to 35° is 11 m:

a) 5 m **b)** 6 m **c)** 6 m

Carol on same side as neither. Distance to 66° is 11 m:

a) 9 m **b)** 11 m **c)** 19 m

Carol on same side as 66° only. Distance to 35° is 11 m:

a) 5 m **b)** 6 m **c)** 2 m

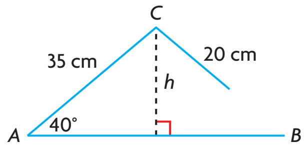
Carol on same side as 66° only. Distance to 66° is 11 m:

a) 28 m **b)** 36 m **c)** 63 m

13. 481 m

14. e.g., Two forest fire stations, *A* and *B*, are 20 km apart. A ranger at station *B* sees a fire 15 km away. If the angle between the line *AB* and the line from *A* to the fire is 21°, determine how far station *A* is from the fire. (two possible triangles)

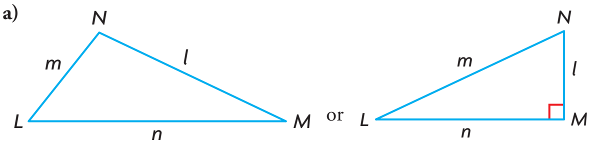
15. **a)** $h = 22.5$ cm, no triangle since $\angle A$ is acute with $a < h$



b) $\angle A = 34.8°$

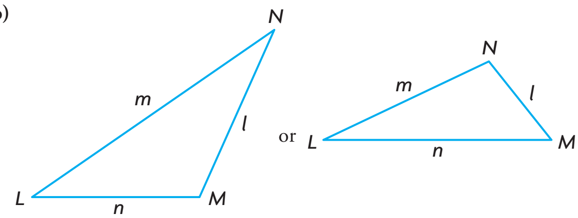
c) $0° < \angle A < 34.8°$

16. **a)**



$l > m$, $\dfrac{\sin L}{l} = \dfrac{\sin M}{m}$, $h = m \sin L$ $l < m$, $l = h$, $l = m \sin L$

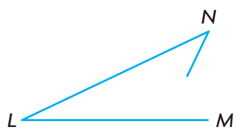
b)



$h = m \sin L$
$m \sin L < l < m$

$h = m \sin L$
$m \sin L < m < l$

c)



$h = m \sin L$
$l < h$

17. **a)** 81.2 cm^2

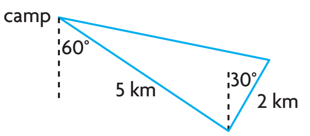
b) 7.5 cm^2

c) 73.7 cm^2

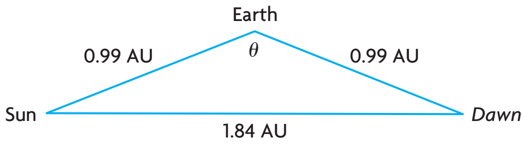
d) e.g., Subtract the area of $\triangle DEF_2$ from $\triangle DEF_1$.

Lesson 4.4, page 193

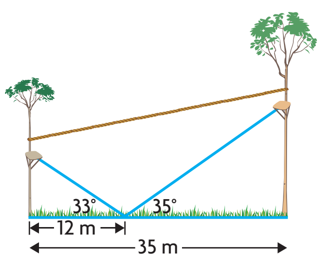
1. a) i) sine law ii) sine law iii) cosine law
   b) i) 51.7 m ii) 140.1° iii) 86.2°
2. tower A: 7.4 km; tower B: 16.5 km
3. 110 m
4. 19.4 ft
5. a) e.g., Bijan is about 5.5 km from camp and should still be in range.



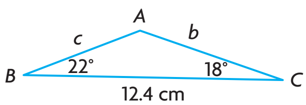
   b) 5.4 km. Yes, he is less than 6 km away.
6. a)



   b) 137°
7. 280 m
8. 12 206 ft
9. e.g., Yes. Bert can use the sine law and the triangle with vertices *A*, *B*, and the base of the tree to determine the distance from *A* to the base of the tree. He can then use the tangent ratio and the 28° angle to determine the height of the tree.
10. Chester's fire department is closer by 13 km.
11. 4410 m
12. e.g., I used a 3-D diagram that was made up of two right triangles. The tangent ratio can be used to determine the distance from Brit to the sailboat. This is the same distance that Tara is from the boat. I would then use the cosine law and the triangle with vertices Brit, Tara, and the sailboat, to determine the angle between the boat as seen from Brit's position.
13. e.g., I assumed that the point 12 m from the base of the smaller tree is between the two trees, and the angle of elevation to the smaller tree is 33°.



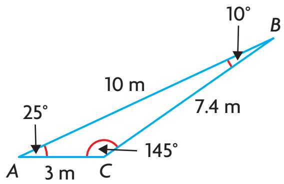
   b) 36 m
14. about 13°
15. e.g., $\angle A = 140°$, $b = 7.2$ cm, $c = 6.0$ cm



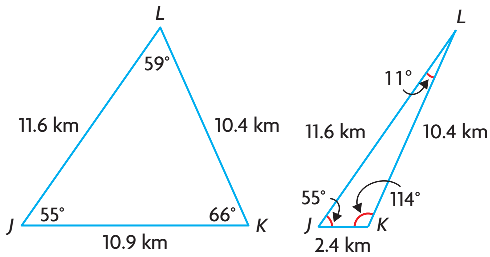
16. a) 4 km from lighthouse A, 13 km from lighthouse B
    b) 3 km
17. 290.2 km

Chapter Self-Test, page 198

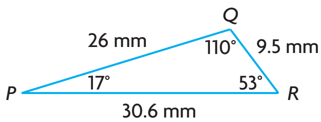
1. a) 76.4°
   b) 66.6°
   c) 0.9° or 179.1°
   d) 138.6°
   e) 30° or 150°
   f) 36.9° or 143.1°
2. a) zero
   b) one



   c) two



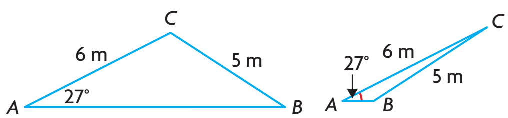
   d) one



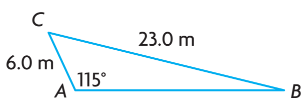
   e) zero
3. 9 ft or 36 ft
4. 75.0 m
5. 4 cm, 11 cm, 12 cm
6. 17 paces or 39 paces
7. 30 m

Chapter Review, page 200

1. e.g., For any obtuse angle θ, $\sin\theta = \sin(180° - \theta)$, $\sin 60° = \sin 120°$; $\cos\theta = -\cos(180° - \theta)$, $\cos 60° = -\cos 120°$; $\tan\theta = -\tan(180° - \theta)$, $\tan 60° = -\tan 120°$.
2. a) 0.8480; 58° c) −0.1736; 80°
   b) 0.8480; 122° d) 0.2679; 165°
3. a) 53.1° b) 80.8°
4. $\angle C = 25°$, $a = 15.5$ cm, $b = 9.5$ cm
5. a) zero
   b) two



   c) one



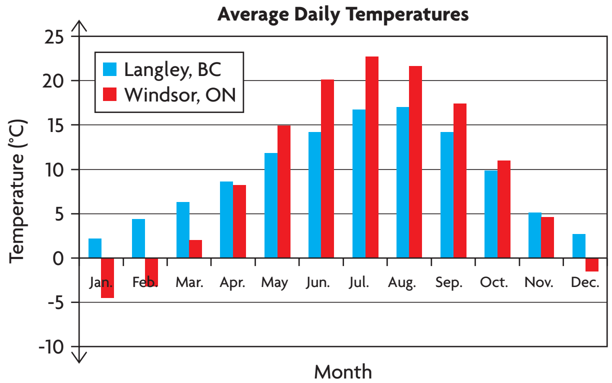
6. a) 1.5 m b) 40.5°
7. a) 4.8 m or 3.5 m b) 1.1 m
8. 1.1 km or 4.5 km

Answers

Chapter 5

Lesson 5.1, page 211

1. **a)**



b)

	Langley, BC (°C)	Windsor, ON (°C)
Range	14.8	27.2
Mean	9.4	9.4
Median	9.2	9.6

c) e.g., The mean temperature for each city is the same, and the medians are close; however, the temperature in Windsor has a much greater range: it gets colder in winter and warmer in summer.

d) e.g., if you were living in one of the locations and moving to the other location

2. **a)**

	Unit 1 Test (%)	Unit 2 Test (%)
Range	24	61
Mean	71.2	71.2
Median	73	73
Mode	73	73

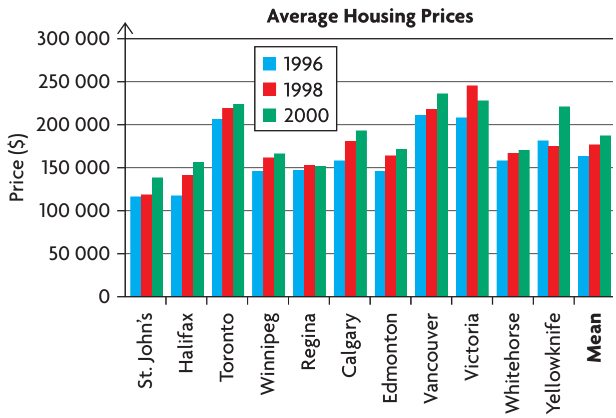
b) e.g., The class performed better on the Unit 1 test because the range of scores was smaller, with the mean, median, and mode being equal.

c) e.g., The modes were not very useful to compare in this context because they only tell me which mark occurred most often, not on which test the class performed better.

3. **a)** e.g.,

	1996 ($)	1998 ($)	2000 ($)
Range	95 567	127 616	98 952
Mean	163 440	176 937	187 434
Median	157 677	167 396	172 503

The data distribution is scattered fairly widely during each year; some cities are much lower or much higher than the mean.

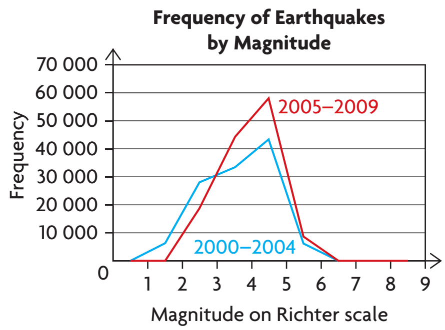


The range between the maximum and minimum average prices in the 11 cities is the greatest in 1998, so that year some prices were much lower than average and some were much higher. Both the mean and the median of the average price in the 11 cities has steadily increased over the 5-year period. In all the cities except for Regina and Victoria, there has been an increase in price over the 5-year period. Also, Yellowknife has had the greatest increase in average price over the 5-year period.

b) e.g., if you were comparing housing costs in cities you are contemplating moving to

Lesson 5.2, page 221

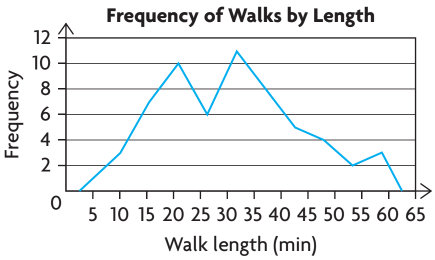
1. **a)**



b) e.g., From 2005 to 2009, there were more earthquakes than from 2000 to 2004. The earthquakes in 2005–2009 tended to be of greater magnitude than those in 2000–2004. 2000–2004 had many more earthquakes that rated less than 3.0, although both periods had roughly the same number of earthquakes that rated more than 7.0.

2. **a)** 10–15 min interval

b) e.g., Most of the data is distributed in the 20–25 min interval and 30–35 min interval.

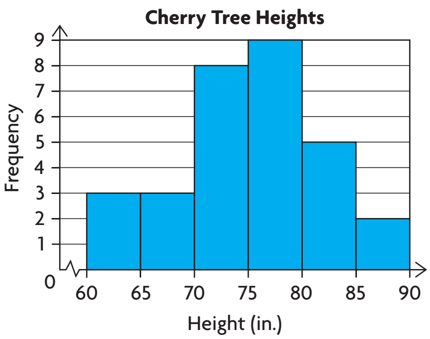


3. e.g.,

a)

Tree Height (in.)	Frequency
60–65	3
65–70	3
70–75	8
75–80	9
80–85	5
85–90	2

b)



c) The range of heights 75–80 inches occurs most frequently. The range of heights 85–90 inches occurs least frequently.

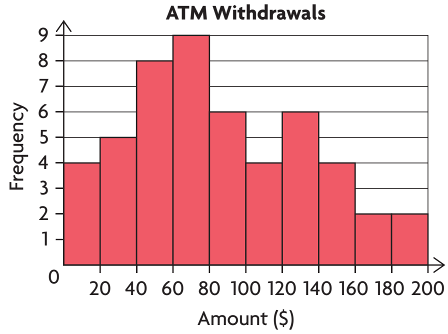
4. e.g.,

a) Most withdrawals are multiples of 20. An interval width of 20 would give a good representation of the distribution of the data.

b)

Withdrawal ($)	Frequency
0–20	4
20–40	5
40–60	8
60–80	9
80–100	6
100–120	4
120–140	6
140–160	4
160–180	2
180–200	2

c)



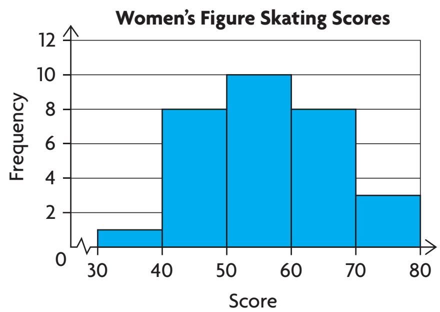
d) There are a lot more withdrawals under $100 than there are over $100. Withdrawals between $40 and $80 are the most frequent. Not many people made withdrawals over $160.

5. e.g,

a)

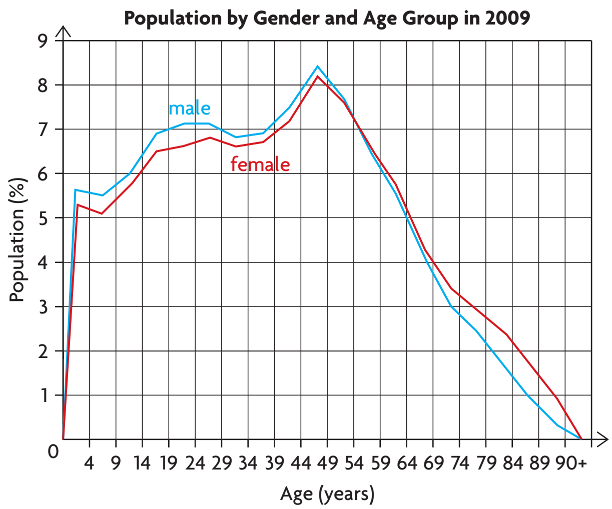
Final Scores	Frequency
30–40	1
40–50	8
50–60	10
60–70	8
70–80	3

b)



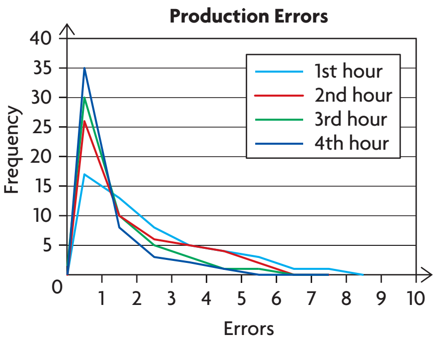
c) No. It shows that three women scored between 70 and 80, but it does not show the range of scores for a top-five placement.

6. **a)**



b) e.g., There are more males than females for all age groups up to 54 years. Starting at age 55, there are more women than men.

7. a)



b) e.g., As the day progresses, the number of errors on a vehicle decreases. Fewer vehicles have large numbers of errors.

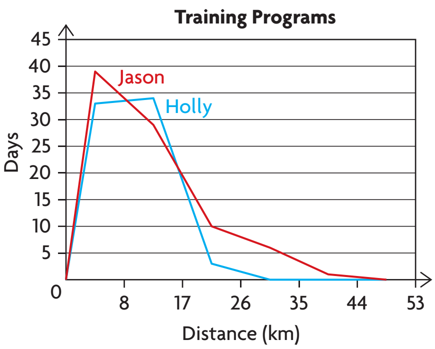
8. e.g.,

a) I chose interval sizes that created five interval spaces that worked for both tables.

Holly's Program	
Kilometres	**Frequency**
0–8	33
8–17	34
17–26	3
26–35	0
35–44	0

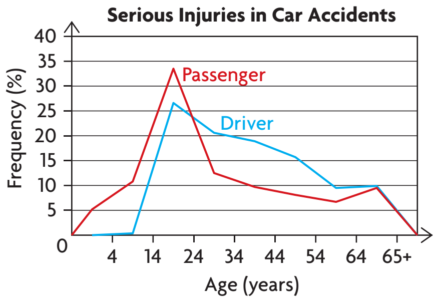
Jason's Program	
Kilometres	**Frequency**
0–8	39
8–17	29
17–26	9
26–35	6
35–44	1

b)



c) Holly's program involves more short distance running, while Jason's program involves more long distance running.

9.



e.g., Younger drivers are involved in more accidents where there are serious injuries. Also, the greatest number of serious injuries for passengers is in the 15 to 24 age group; perhaps these passengers were in the same accidents as the young drivers who were seriously injured (i.e., out driving with friends).

10. e.g., Using intervals of equal width enables you to see the distribution more easily and to compare the data more effectively.

11. e.g.,

a) Grouping raw data into intervals makes it easier to interpret the data accurately and to see the distribution. It also makes the data more manageable.

b) Histograms compare data intervals side by side using bars, while frequency polygons compare data intervals using lines. Frequency polygons are useful when comparing two or more sets of data, because you can easily combine them on the same graph, making it easier to see differences or similarities in the data sets.

12. e.g.,

a) 5116. I eliminated all rows in the table with a frequency of 0. Then I determined the midpoint for each remaining interval. Next, I multiplied the frequency of each row by the midpoint to estimate the total population for each row. I determined the mean of the products.

b) 3115. The city with the median population is the 76th one, which occurs in the first row of the table. The 76th city out of 122 might have a population of

$$\frac{76}{122} \cdot 5000 = 3114.754\ldots \text{ or } 3115.$$

c) I assumed that 61 cities would be below 2500 and 61 cities would be between 2500 and 5000. Since 75 cities have a population of less than 1700, the estimates for both the mean and the median are higher than they should be.

Lesson 5.3, page 233

1. a), b) class A: 14.27; class B: 3.61

c) Class B has the most consistent marks over the first five tests since it has the lowest standard deviation.

2. mean: 130.42 points; standard deviation: 11.51 points

3. a) mean: 130.36 points; standard deviation: 12.05 points

b) e.g., Ali's mean and standard deviation are close to his team's. He is an average player on his team.

4. e.g.,

a) The mean number of beads in company B's packages is much less consistent than the mean number of beads in company A's packages.

b) company A

5. Group 1: mean: 71.9 bpm; standard deviation: 6.0 bpm
Group 2: mean: 71.0 bpm; standard deviation: 4.0 bpm
Group 3: mean: 70.4 bpm; standard deviation: 5.7 bpm
Group 4: mean: 76.9 bpm; standard deviation: 1.9 bpm
Group 3 has the lowest mean pulse rate. Group 4 has the most consistent pulse rate.

6. a) Diko b) Nazra
7. a) mean: 10.5 TDs; standard deviation: 5.6 TDs
   b) e.g., He probably played fewer games in his rookie year (his first year) and his last year.
   c) mean: 11.7 TDs; standard deviation: 5.2 TDs
   d) The mean is higher and the standard deviation is lower.
8. a) mean: 1082 yards gained; standard deviation: 428.8 yards gained
   b) Allen Pitts
9. a) Fitness Express: mean: 18.3 h; standard deviation: 4.9 h
   Fit For Life: mean: 19.1 h; standard deviation: 5.3 h
   b) Fitness Express
10. Jaime's mean travel time is about 21.2 minutes and her standard deviation is 3.5 minutes. Since her mean time is more than 20 minutes, Jaime will lose her job.
11. yes; mean: 45.0 calls; standard deviation: 7.1 calls
12. a)

	Games Played	Goals	Assists	Points
Mean	57.5	12.5	16.1	28.6
Standard Deviation	11.4	11.6	13.1	24.5

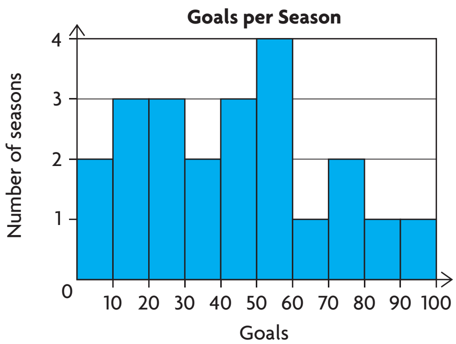
   b) e.g., The standard deviation should decrease for games played and should increase for goals, assists, and points.
   c)

	Games Played	Goals	Assists	Points
Mean	60.1	13.4	17.2	30.7
Standard Deviation	8.7	11.8	13.3	24.9

   d) e.g., The means and standard deviations increased and decreased as I predicted.
   e) e.g., The statement is true for data, and for means because we can add fractions with the same denominator together. However, standard deviations cannot be added because of how they are calculated.
13. e.g., One twin is more consistent, while the other is less consistent, resulting in the same mean (85.0%) with different standard deviations (2.6%, 12.0%).
   Jane's scores: 80%, 85%, 82%, 87%, 86%, 84%, 87%, 85%, 85%, 89%
   Jordana's scores: 78%, 92%, 99%, 64%, 72%, 82%, 77%, 95%, 98%, 93%
14. a) group A: mean: 8.56 s; standard deviation: 7.99 s
   group B: mean: 5.55 s; standard deviation: 4.73 s
   b) yes; group B (the group given visual information)

Mid-Chapter Review, page 239

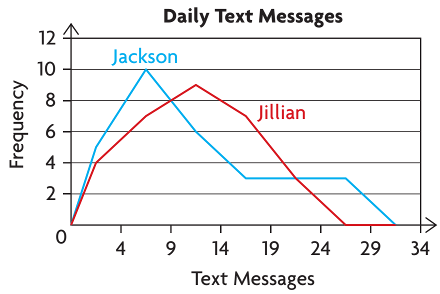
1. Paris: 15.6 °C; Sydney: 17.8 °C
2. e.g., Wayne Gretzky tended to score between 0 and 60 goals per season. He infrequently scored more than that.



3. e.g.,
   a) 5
   b)

Text Messages	Jackson	Jillian
0–4	5	4
5–9	10	7
10–14	6	9
15–19	3	7
20–24	3	3
25–29	3	0

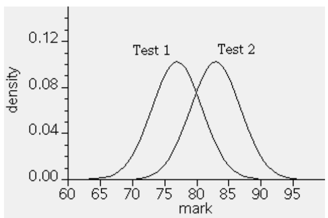
   c)

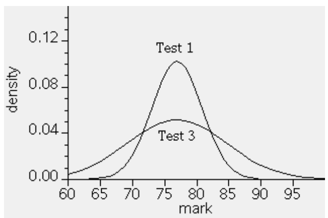


4. Jackson: mean: 11.6 messages; standard deviation: 7.4 messages
   Jillian: mean: 11.7 messages; standard deviation: 6.0 messages
   e.g., Jillian and Jackson send about the same number of text messages, but Jillian is more consistent with her daily amount.
5. a) range: $42.00; mean: $21.95; standard deviation: $8.24
   b) range: $15.00; mean: $21.35; standard deviation: $4.54
   c) Removing the greatest and least amounts reduces the standard deviation.
6. females: mean: $27 391.30; standard deviation: $7241.12
   males: mean: $41 614.79; standard deviation: $19 542.92
   e.g., Males tend to have larger salaries, but their salaries are less consistently close to the mean, suggesting a greater range.

Lesson 5.4, page 251

1. a) 47.5% b) 15.85% c) 0.15%
2. a)



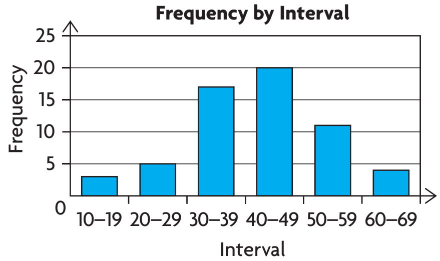


b) e.g., Test 1 and test 2 have different means, but the same standard deviation. Test 1 and test 3 have the same mean, but different standard deviations.

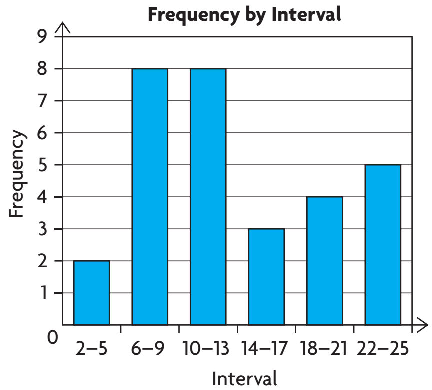
c) test 1: 84.8%; test 2: 79.1%; test 3: 99.2%

3. e.g.,

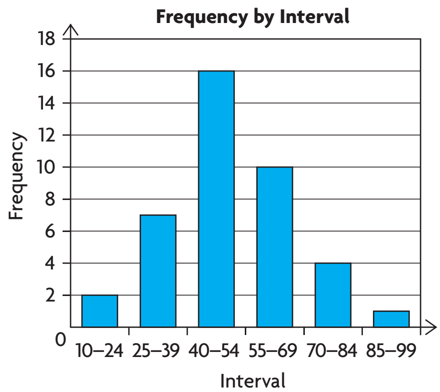
a) Yes. A graph of the data has a rough bell shape.



b) No. A graph of the data does not have a bell shape.



c) Yes. A graph of the data has a rough bell shape.



4. **a)** mean: 104.5 min; standard deviation: 22.3 min

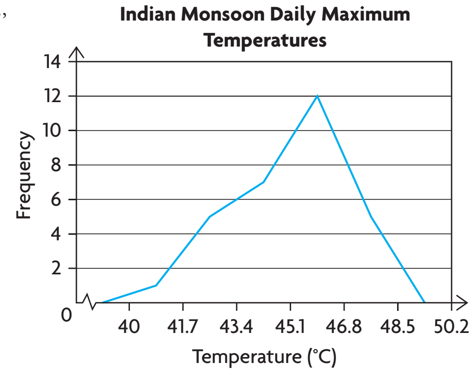
b) e.g.,

Movie Length (min)	Frequency
59.5–82.0	3
82.0–104.5	33
104.5–127.0	7
127.0–149.5	3
149.5–172.0	3
172.0–194.5	1

c) e.g., No. 80% of the data is within 1 standard deviation of the mean.

5. **a)** **i)** mean: 45.2 °C; median: 45.5 °C; standard deviation: 1.7 °C

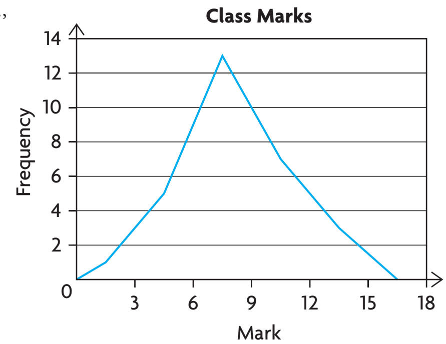
ii) e.g.,



iii) e.g., The median is close to the mean, but the frequency polygon is not symmetric around the mean, so the data is not normally distributed.

b) **i)** mean: 8.6; median: 8; standard deviation: 2.8

ii) e.g.,

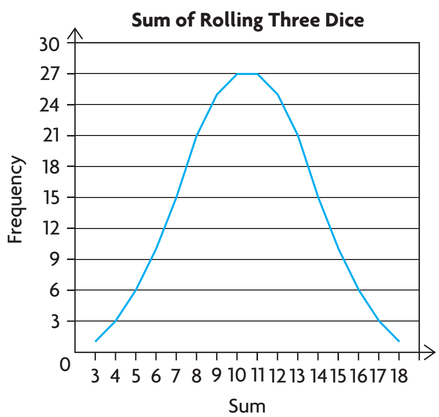


iii) e.g., The shape of the graph is roughly symmetrical with one peak in the middle tapering off to either side. The mean and median are fairly close to each other. The distribution is approximately normal.

6. about 3 years

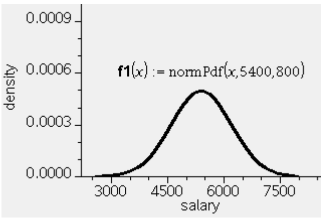
7. **a)** mean: 10.5; standard deviation: 2.96

b) e.g.,

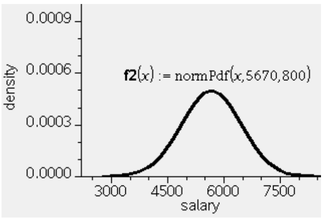


c) e.g., Yes, when you determine the percent of data in the various sections of the graph, they match the percent of data in a normal distribution.

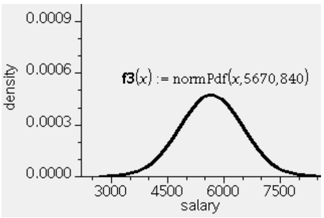
8. **a)**



b)



c)



9. e.g.,

a) Yes, when I determine the percent of the data within 1, 2, 3, and 4 standard deviations of the mean, they agree with the percents for a normal distribution.

b) The mean is 72.25, the median is 72, and the mode 73. The values are close together, so the golf scores appear to be normally distributed.

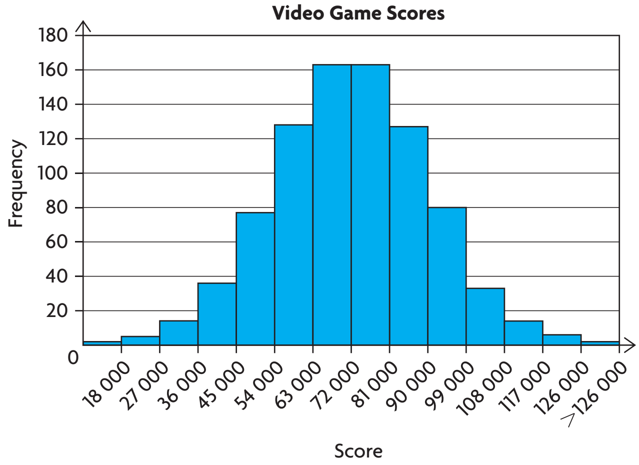
10. 2.5%, or about 3 dolphins

11. **a)** 44.6 kg–99.0 kg

b) 31.0 kg–112.6 kg

c) e.g., Julie assumed that the masses of North American men and women is normally distributed about the mean. However, men and women have different mean masses.

12. **a)** Yes



b) Yes; mean: 72 010 points; standard deviation: 18 394 points. The percent of scores within 1, 2, and 3 standard deviations are very close to the expected values for a normal distribution:

$\mu \pm 1\sigma = 68.35\%$

$\mu \pm 2\sigma = 94.94\%$

$\mu \pm 3\sigma = 99.53\%$

13. **a)** 68%, or about 41 dogs **c)** 99.7%, or about 60 dogs

b) 95%, or about 57 dogs **d)** 50%, or about 30 dogs

14. mean: 482 kg; standard deviation: 17 kg

15. e.g., The 10 students could all have the highest marks in the class, so they would not be normally distributed.

16. e.g., No. The male dog would have been over 10 standard deviations heavier than average, and the female dog would have been over 13 standard deviations lighter than average. These masses are improbable.

Lesson 5.5, page 264

1. **a)** 4 **c)** −1.92

b) −0.75 **d)** 2.455…

2. **a)** 89.25% **c)** 98.50%

b) 0.94% **d)** 26.11%

3. **a)** 6.88% **b)** 75.18%

4. **a)** −1.28 **c)** 0.25

b) 1.28 **d)** −0.25

5. **a)** 1.892… **c)** 0.505…

b) −2.6875 **d)** 1.666…

6. **a)** 71.23% **c)** 0.14%

b) 3.92% **d)** 99.16%

7. **a)** 91.15% **c)** 24.83%

b) 0.43% **d)** 99.92%

8. **a)** 39.95% **b)** 89.90%

9. **a)** −0.439… **b)** 0.841…

10. **a)** English: 2.352… Math: 3.148…

b) Math

c) e.g., the job market, her preferences, whether absolute or relative marks are more important for university applications

11. 92.70%

12. water walking

13. **a)** 90.60% or 91.24%, depending on method used

b) 3.11% or 3.14%, depending on method used

c) 0.88%; e.g., Someone might want to see if the percentage of high-school age mothers with young children is decreasing or increasing. Someone might use this data to justify funding for social programs targeting this group.

14. mean: 180 cm; standard deviation: 15.6 cm

15. a) 0.38% b) 37.81% c) 5.29

16. a) about 2 b) 10.56%

17. 50 months, or round down to 4 years

18. 76%

19. e.g., For the high-priced car, the z-score is 0.5, which means that about 69% of the repairs will be less than \$3000; thus, 31% of the repairs will be more than \$3000. For the mid-priced car, the z-score is 1.25, which means that about 89% of the repairs will be less than \$3000; thus, 11% of the repairs will be more than \$3000.

20. a) 131 b) 140 c) at least 108

21. A z-score is a value that indicates the number of standard deviations of a data value above or below the mean. It is calculated by subtracting the mean from the data value, and then dividing by the standard deviation. Knowing the z-score of two or more pieces of data in different data sets allows you to compare them, which is useful for making decisions.

22. a) 5.02 kg
b) 62.1%; e.g., No; too many bags will have more than 5 kg of sugar.

23. a) mean: 150; standard deviation: about 9.6
b) 1.1%

24. e.g., If the ABC Company wants its process to meet 6-Sigma standards, that is, to reject fewer than 1 bungee cord per 300 produced, what standard deviation does the company need to have in its manufacturing process? Answer: The ABC Company needs to reduce its standard deviation to 1.0 cm if it wants to reject only 0.33% of bungee cords.

Lesson 5.6, page 274

1. a) 95%
b) 77.9%–84.1%
c) 26.1 million to 28.2 million

2. a) 540.1 g to 543.9 g
b) 50: 3.9 g; 100: 2.7 g; 500: 1.2 g

3. a) 90%
b) 60.6%–67.4%
c) about 19–22 students

4. a) With 95% confidence, it can be said that 78.8% to 83.2% of Canadians support bilingualism in Canada and that they want Canada to remain a bilingual country.
b) e.g., I disagree with Mark. Without having more information about how the poll was conducted, it is impossible to tell if the poll was flawed.

5. a) 54.9%–61.1%
b) e.g., Swift Current, Saskatchewan, has a population of 16 000 residents. Between 8784 and 9776 people would have answered the question correctly.

6. a) 99%; 84.7% to 93.3%
b) 19 904 500 to 21 925 500

7. e.g.,
a) The Canadian Press Harris-Decima surveyed Canadians in early 2010 to find out how people felt about a proposed rewording of "O Canada." The poll found that 74% of Canadians opposed the rewording, with a margin of error of 2.2 percentage points, 19 times out of 20.
b) 71.8%–76.2%
c) With such a high percent of Canadians polled opposing the rewording, and the relatively tight confidence interval, I agree with the conclusion of the poll.

8. a) confidence interval: 174.8 g to 175.2 g
margin of error: $\pm$0.2 g
b) 135
c) 55
d) 78

9. a) With 90% confidence, it can be said that 49.5% to 58.5% of post-secondary graduates can be expected to earn at least \$100 000/year by the time they retire.
b) With 99% confidence, it can be said that 60.9% to 65.1% of online shoppers search for coupons or deals when shopping on the Internet.
c) With 95% confidence, it can be said that Canadians spend an average of 17.5 h to 18.7 h online, compared to 16.3 h to 17.5 h watching television per week.
d) With 95% confidence, it can be said that 36% to 42% of decided voters will not vote for the political party in the next election.

10. a) As the sample size increases, a larger proportion of the population is sampled, making the results more representative of the population, therefore reducing the margin of error.
b) For the confidence level to increase, the size of the confidence interval must increase; therefore, the more confident you are that a value falls within the range, the more the margin of error increases.

11. a)

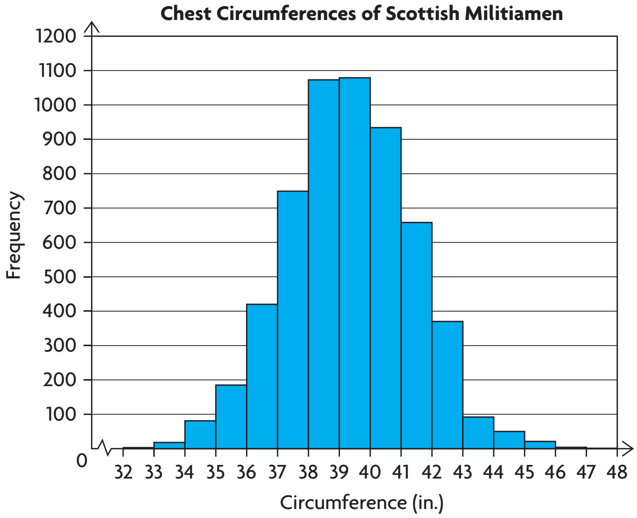
Sample Size	Pattern	Margin of Error
100		9.80%
400	$\sqrt{\frac{100}{400}} = \frac{1}{2}$	$9.80\% \cdot \frac{1}{2} = 4.90\%$
900	$\sqrt{\frac{400}{900}} = \frac{2}{3}$	$4.90\% \cdot \frac{2}{3} = 3.27\%$
1600	$\sqrt{\frac{900}{1600}} = \frac{3}{4}$	$3.27\% \cdot \frac{3}{4} = 2.45\%$
2500	$\sqrt{\frac{1600}{2500}} = \frac{4}{5}$	$2.45\% \cdot \frac{4}{5} = 1.96\%$
3600	$\sqrt{\frac{2500}{3600}} = \frac{5}{6}$	$1.96\% \cdot \frac{5}{6} = 1.63\%$

b) i) 1.40% ii) 2.19%
c) e.g., The margin of error gets smaller at a much faster rate than the sample size grows. Therefore, a relatively small sample is needed to get a small margin of error.

Chapter Self-Test, page 277

1. a) 1999: mean: 43.4 in.; standard deviation: 3.0 in.
2011: mean: 67.8 in.; standard deviation: 5.6 in.
b) The heights for 2011 have a greater standard deviation. Children are much closer in height than teenagers, which is why there is greater deviation in the teenagers' heights.

2. a) e.g., The graph of the data shows a normal distribution.



b) 1.06

3. e.g., Edmonton's temperature is lower on average, but less consistently close to the mean.

4. a) If the poll was conducted with a random sample of 1009 Canadians 100 times, you can be confident that 95 times the results would be that
   - 84.9% to 91.1% of people would say that the flag makes them proud of Canada
   - 76.9% to 83.1% of people would say that hockey makes them proud of Canada
   - 40.9% to 47.1% of people would say that our justice system makes them proud of Canada

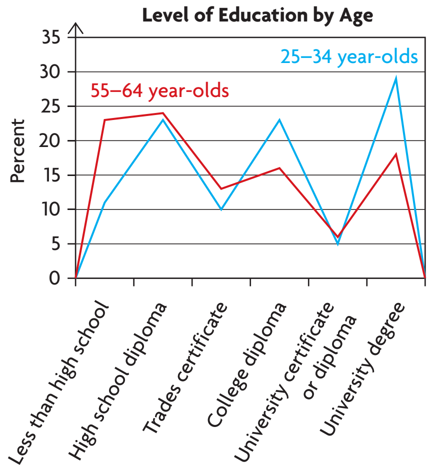
   b) 26 160 611 to 28 269 789 people

   c) The margin of error would increase since the confidence level that would be used in the new poll increased to 99% from 95%.

Chapter Review, page 280

1. Twila: mean: 46.5 min; range: 25 min
   Amber: mean: 45.5 min; range 75 min
   e.g., Each girl spent about the same amount of time on homework every day, but Amber spends a greater range of times on homework than Twila.

2. e.g., More people aged 25 to 34 years old have higher levels of education.



3. a) e.g., Twila's data will have the lowest standard deviation. The numbers are closer to the mean.

   b) Twila: 7.8 min; Amber: 26.9 min; yes

4. a)

Level of Education	Mean ($1000)	Standard Deviation ($1000)
No Diploma	18.8	2.6
High School	23.8	4.1
Post-Secondary	36.8	5.5

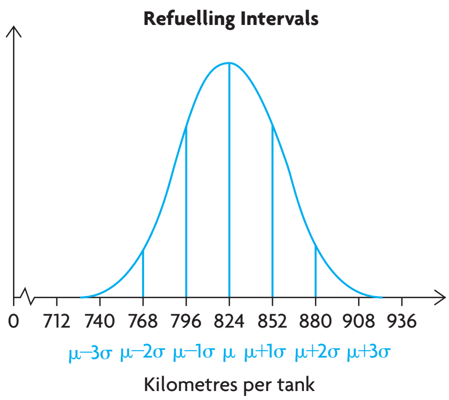
   b) post-secondary

   c) post-secondary

5. bag of sunflower seeds

6. female bear

7. a)



   b) 68%

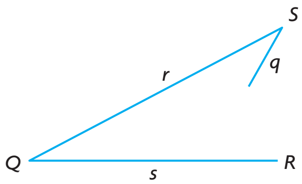
   c) 16%

   d) 768 km and 880 km

8. a) mean: 36.9 °C; standard deviation: 0.4 °C
b) e.g., Yes. About 69% of the data is within one standard deviation of the mean, about 94% of the data is within two standard deviations of the mean, and about 99% of the data is within three standard deviations of the mean, which is close to the percents expected for a normal distribution.
9. 10.6%
10. Computers For All
11. a) Internet: 60.5% to 63.3%
Friends/family: 67.5% to 70.3%
Health line: 16.5% to 19.3%
b) Internet: 208 725 to 218 385 people
Friends/family: 232 875 to 242 535 people
Health line: 56 925 to 66 585 people
12. a) The margin of error of company A is larger than the margin of error of company B.
b) Company A's sample was larger than company B's sample.

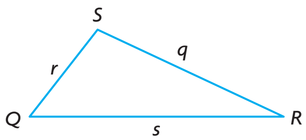
Cumulative Review, Chapters 3–5, page 287

1. a) $x = 12.6$ cm c) $d = 9.4$ cm, $e = 12.7$ cm
b) $p = 7.6$ m d) $m = 16.9$ m
2. a) 40° c) 43°
b) 104° d) 117°
3. $\angle H = 66.7°$, $h = 11.1$ cm, $\angle J = 65.3°$ or
$\angle H = 17.3°$, $h = 3.6$ cm, $\angle J = 114.7°$
4. $d = 12.2$ cm, $\angle E = 44.2°$, $\angle F = 77.7°$
5. 3.5 km, N39.0°W
6. 261 m
7. a)



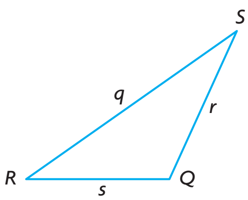
$h = r \sin Q$, $q < h$

b)



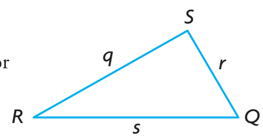
$q > r$ or $q = r$, $\frac{\sin Q}{q} = \frac{\sin R}{r}$, $h = r \sin Q$

c)



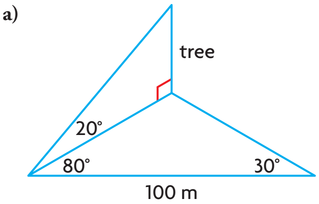
$h = r \sin Q$
$r \sin Q < r < q$
$\angle Q$ obtuse

or



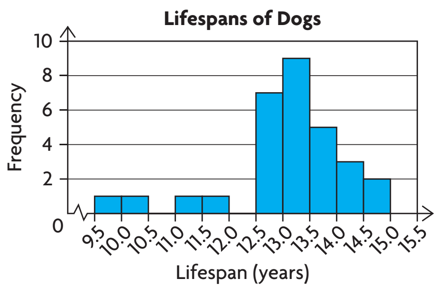
$h = r \sin Q$
$r \sin Q < r < q$
$\angle Q$ acute

8. a)



b) 19.4 m
9. e.g.,
a)

Lifespan (years)	Frequency
9.5–10.0	1
10.0–10.5	1
10.5–11.0	0
11.0–11.5	1
11.5–12.0	1
12.0–12.5	0
12.5–13.0	7
13.0–13.5	9
13.5–14.0	5
14.0–14.5	3
14.5–15.0	2

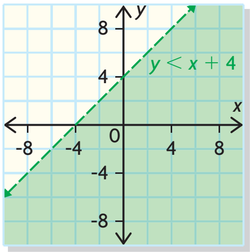


b) No
c) range: 5.2 years; standard deviation: 1.1 years. e.g., The data does not deviate very much from the mean.
10. e.g., Winnipeg and Whitehorse have approximately the same temperature in January, but the temperature varies more in Whitehorse.
11. a) 97.7% b) 2.3%
12. a) 2.3% b) 15.7%
13. a) 3.1%
b) 95%
c) i) 20.0%–26.2% ii) 9.5%–15.7%
14. e.g.,
a) The confidence level decreases as the margin of error decreases because we can be less certain that the true mean is in the range specified.
b) The confidence level decreases as the sample size decreases because there is more chance that the mean of a particular sample will fall outside the confidence interval.

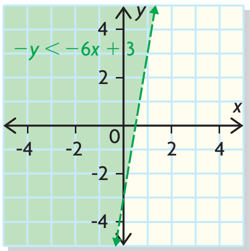
Chapter 6

Lesson 6.1, page 303

1. **a)**



b)

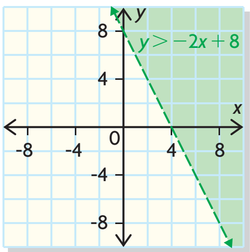


2. **a)** dashed
b) shaded above boundary
c) **i)** No **ii)** No **iii)** Yes

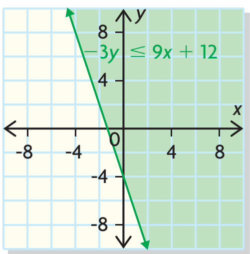
3. **a)** b represents the number of hours Betsy usually works; f represents the number of hours Flynn usually works.
b) real numbers; f and b must be greater than zero.
c) **i)** dashed green line along $3b + 2f = 25$
ii) Below; (0, 0) is a solution.
iii) No; time cannot be negative.
d) A solution represents a possible combination of hours usually worked by Betsy and Flynn that satisfy the given conditions.

4. **a)** ii **b)** i **c)** iii

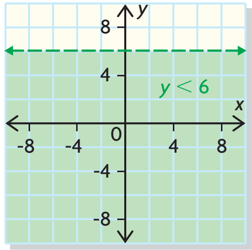
5. **a)**



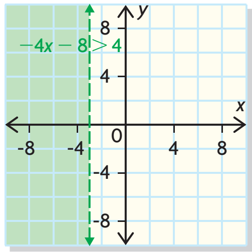
b)



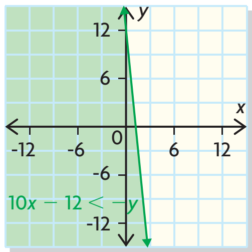
c)



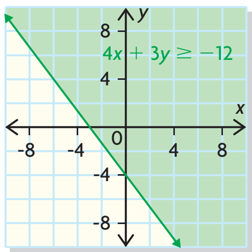
d)



e)

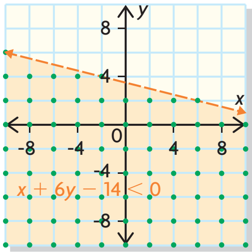


f)

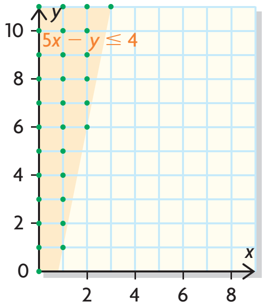


6. **a)** no solution

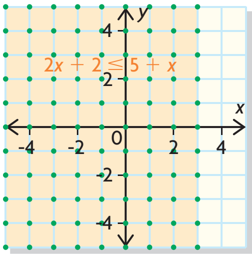
b)



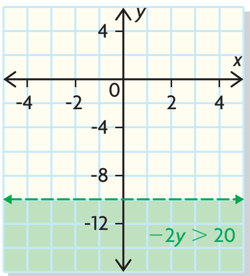
c)



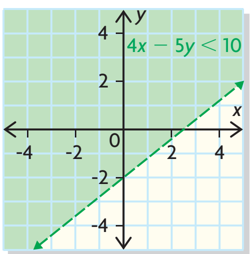
d)



e)



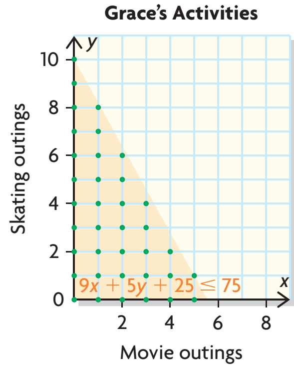
f)



7. **a)** Let x represent the number of movies Grace sees. Let y represent the number of times Grace goes skating.
$\{(x, y) \mid 9x + 5y + 25 \leq 75, x \in W, y \in W\}$

b) The variables must be whole numbers. $x \in W, y \in W$

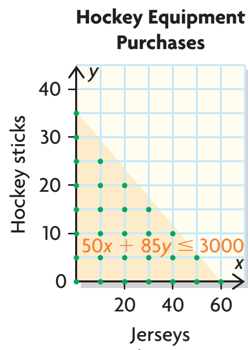
c)



i) e.g., see 3 movies and go skating 4 times
ii) e.g., see 5 movies and go skating once
iii) e.g., see 3 movies and go skating 6 times

8. **a)** Let x represent the number of jerseys. Let y represent the number of sticks.
$\{(x, y) \mid 50x + 85y \leq 3000, x \in W, y \in W\}$

b)

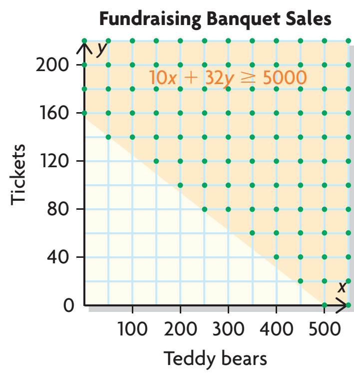


c) e.g., Eamon can buy 20 practice jerseys and 20 sticks for his team for \$2700. It's reasonable to have a few extra jerseys and a few extra sticks.

9. **a)** Let x represent the number of teddy bears sold. Let y represent the number of tickets sold.
$\{(x, y) \mid 10x + 32y \geq 5000, x \in W, y \in W\}$

b) The variables must be whole numbers. $x \in W, y \in W$

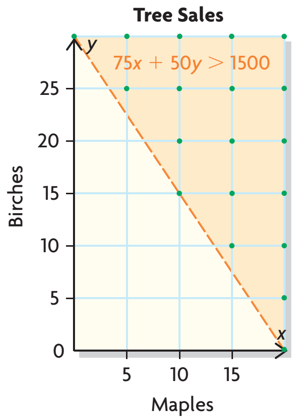
c)



i) not a solution
ii) Yes, this is a solution.
iii) not a solution

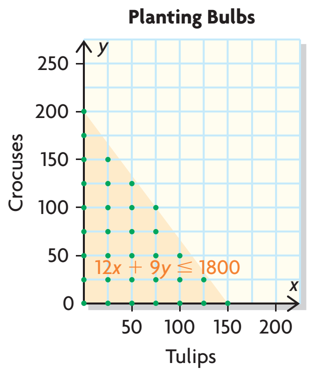
10. **a)** Let x represent the number of maple trees sold. Let y represent the number of birch trees sold.
$\{(x, y) \mid 75x + 50y > 1500, x \in W, y \in W\}$
The variables must be whole numbers. $x \in W, y \in W$

b)



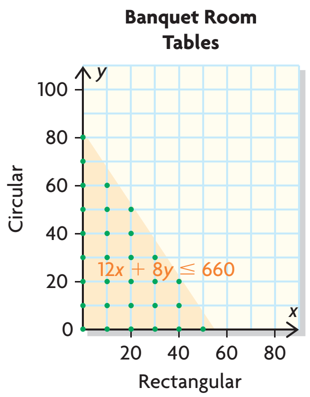
c) **i)** Yes, because (13, 13) is in the solution region.
ii) No, because (14, 9) lies on the dashed boundary and is not included in the shaded region; the point (9, 14) is also not in the solution region.

11. **a)** The boundary would be stippled. The number of plants would have to be a discrete number.
b) Let x be the number of tulips planted. Let y be the number of crocuses planted.



e.g., 75 tulips and 100 crocuses

12. **a)** Let x represent the number of rectangular tables. Let y represent the number of circular tables.
$\{(x, y) \mid 12x + 8y \leq 660, x \in W, y \in W\}$

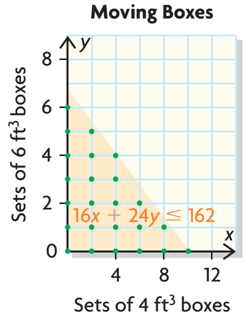


b) The organizers can use 33 of each type of table to give 660 seats.

13. **a)** e.g., The boundary is a straight line and a half plane is shaded.
b) e.g., Only discrete whole number points in the shaded region and on the axes are included; the domain and range is the set of whole numbers; the linear inequality is

$\{(x, y) \mid y > -\frac{7}{3}x + 7, x \in W, y \in W\}$

14. **a)**

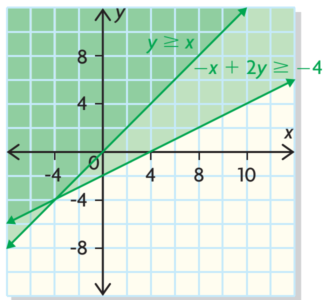


b) 7 sets of the larger boxes

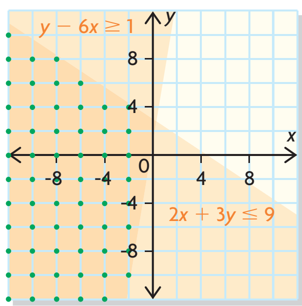
Lesson 6.2, page 307

1. **a)** $x \in R, y \in R$
b) $x \in I, y \in I$
c) $x \in I; y \in I$

2. **a)**

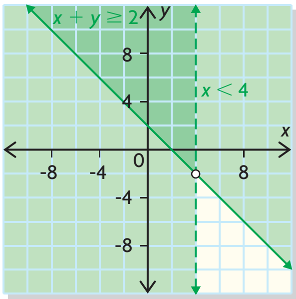


b)

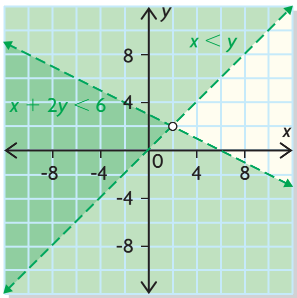


Lesson 6.3, page 317

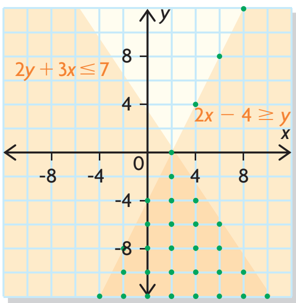
1. a) e.g., (2, 2)



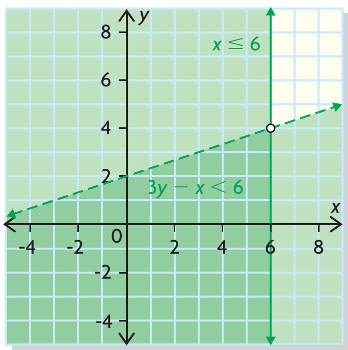
b) e.g., (−4, 2)



c) e.g., (2, −4)



2. a) The solution set is the overlapping region with a boundary along the line $x = 6$ and points along the boundary are included. The other boundary is $3y - x = 6$, but points along it are not included.



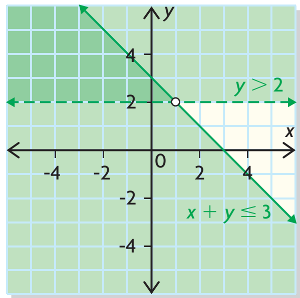
b) i) No **ii)** No **iii)** Yes **iv)** No

3. a) boundary $y \geq -2x$ included; boundary $-3 < x - y$ not included; intersection not included

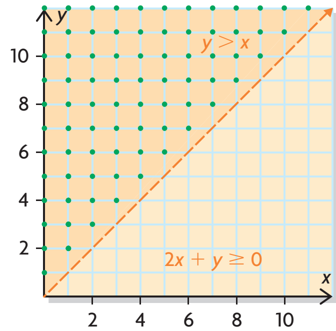
b) boundary $x + y \leq -2$ points with integer coordinates included; boundary $2y \geq x$ points with integer coordinates included; intersection point does not have integer coordinates and is not included

c) boundary $x + 3y \geq 0$ points with integer coordinates included; boundary $x + y > 2$ not included; intersection not included

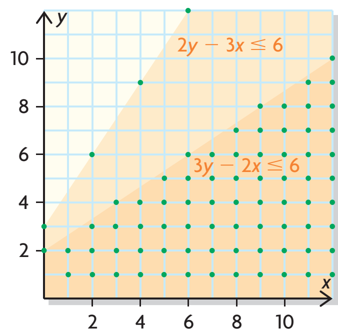
4. a) e.g., (−2, 3)



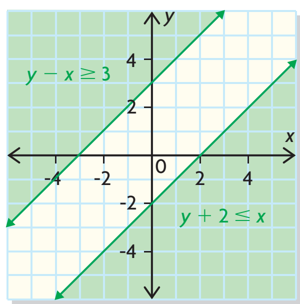
b) e.g., (2, 6)



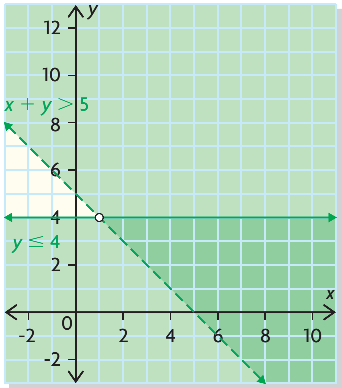
c) e.g., (8, 2)



d) no solution



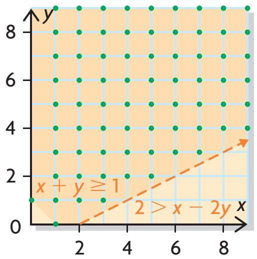
5. **i)** **a)**



b) region below and on $y \leq 4$, and above $x + y > 5$

c) e.g., (6, 3)

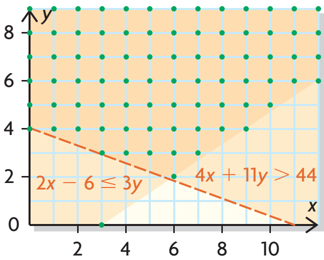
ii) **a)**



b) points with natural number coordinates above the line $2 = x - 2y$, and on and above the line $x + y = 1$

c) e.g., (2, 2)

iii) **a)**



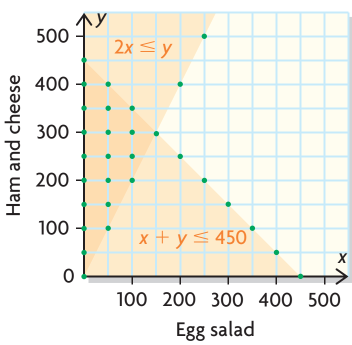
b) points with whole number coordinates above $4x + 11y = 44$ and above or on $2x - 6 = 3y$

c) e.g., (6, 4)

6. **a)** Let x represent the number of egg salad sandwiches. Let y represent the number of ham and cheese sandwiches.
$\{(x, y) \mid x + y \leq 450, x \in W, y \in W\}$
$\{(x, y) \mid 2x \leq y, x \in W, y \in W\}$

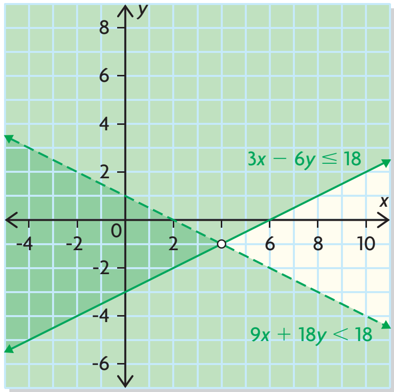
b) The variables must be whole numbers. $x \in W, y \in W$

c) **Cafeteria Sandwiches**



d) e.g., 50 egg salad and 300 ham and cheese; 100 egg salad and 200 ham and cheese

7. **a)** e.g., possible solution: $(0, -3)$



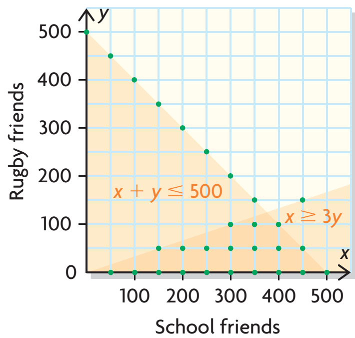
b) **i)** No **ii)** No **iii)** Yes **iv)** No **v)** Yes **vi)** Yes

8. **a)** Let x represent the number of school friends. Let y represent the number of rugby friends.
$\{(x, y) \mid x + y \leq 500, x \in W, y \in W\}$
$\{(x, y) \mid x \geq 3y, x \in W, y \in W\}$

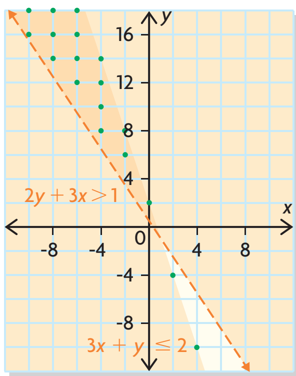
b) The variables must be whole numbers. $x \in W, y \in W$

c) e.g., 200 school friends and 50 rugby friends; 350 school friends and 100 rugby friends

Social Network Friends



9. e.g., $(-3, 8)$, $(-4, 10)$



10. e.g., 28 songs for young listeners and 4 songs for older listeners; 18 songs for young listeners and 8 songs for older listeners; 24 songs for young listeners and 6 songs for older listeners

11. e.g.,

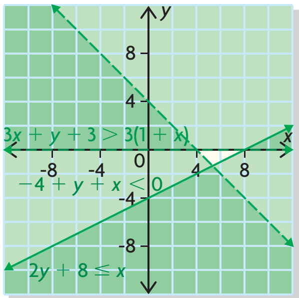
a) $\{(x, y) \mid x + y \geq 3, x \in \mathrm{R}, y \in \mathrm{R}\}$

$\{(x, y) \mid 2x - y \leq 4, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid x + y > 7, x \in \mathrm{W}, y \in \mathrm{W}\}$

$\{(x, y) \mid 2x - y < 4, x \in \mathrm{W}, y \in \mathrm{W}\}$

12. no solution



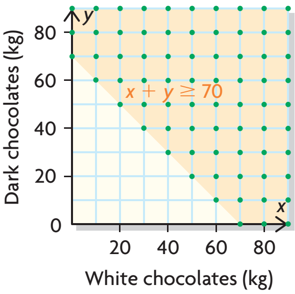
Mid-Chapter Review, page 323

1. **a)** Let x represent the number of kilograms of white chocolates. Let y represent the number of kilograms of dark chocolates.

$x + y \geq 70$

b) e.g., 40 kg white, 50 kg dark; 20 kg white, 60 kg dark; 50 kg white, 30 kg dark

School Fundraiser Sales



2. **a)** e.g., The variables are real numbers. The points below the boundary are part of the solution region. The points on the boundary are not part of the solution region. The inequality is $y < 2x + 2$.

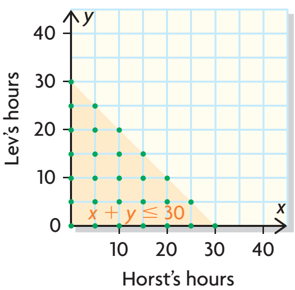
b) **i)** Yes **ii)** Yes **iii)** No **iv)** No

3. **a)** Let x represent the number of hours Horst volunteers. Let y represent the number of hours Lev volunteers.

$\{(x, y) \mid x + y \leq 30, x \in \mathrm{W}, y \in \mathrm{W}\}$

b) e.g., Horst 10 h and Lev 5 h; Horst 5 h and Lev 20 h; Horst 15 h and Lev 7 h

Volunteer Hours



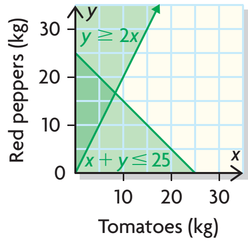
4. **a)** Let x represent the number of kilograms of tomatoes. Let y represent the number of kilograms of red peppers.

$x + y \leq 25$

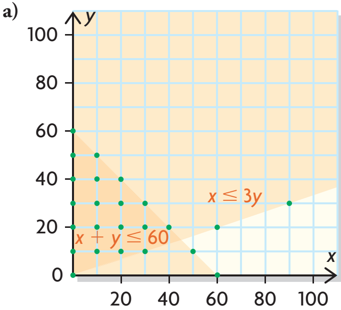
$y \geq 2x$

b) e.g., 5 kg tomatoes and 15 kg peppers; 3 kg tomatoes and 18 kg peppers; 2 kg tomatoes and 10 kg peppers

Red Pepper and Tomato Soup



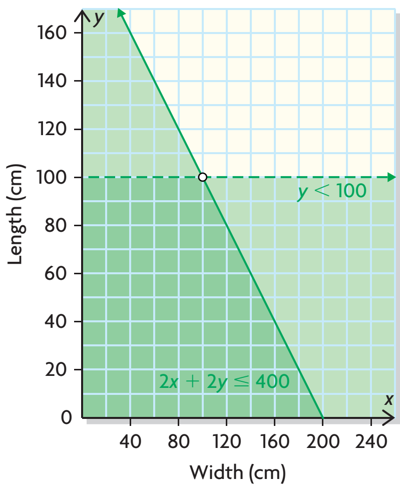
5. **a)**



b) The animal shelter can accommodate up to and including 60 dogs (x) and cats (y) in total. There are, at most, cages for three times as many dogs as cats. How many dogs and cats can be accommodated?

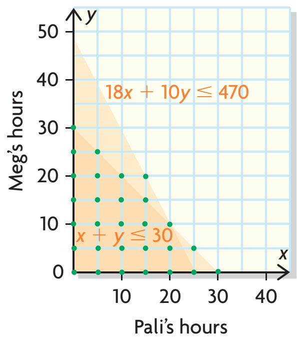
6. e.g., length 75 cm and width 25 cm; length 60 cm and width 20 cm; length 70 cm and width 30 cm

Flag Dimensions



7. **a)** e.g., 20 hours for Pali and 10 hours for Meg; 15 hours each

Work Hours



b) **i)** The solution region will be similar in shape but smaller, so the possible combinations will be reduced.

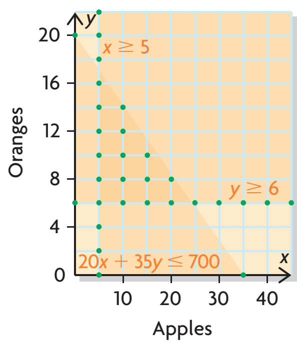
ii) The solution region will be reduced to a quadrilateral region between the x-axis, $18x + 10y \leq 470$, $x + y \leq 30$, and $2y \leq x$.

Lesson 6.4, page 330

1. **a)** number of apples and number of oranges; whole numbers

b) Let x represent the number of apples. Let y represent the number of oranges.
$\{(x, y) \mid x \geq 5, x \in W, y \in W\}$
$\{(x, y) \mid y \geq 6, x \in W, y \in W\}$
$\{(x, y) \mid 20x + 35y \leq 700, x \in W, y \in W\}$

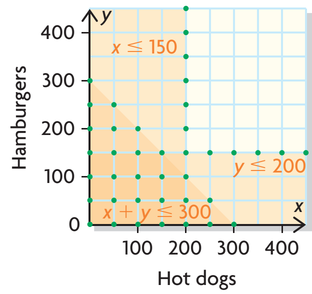
c) **Fruit Basket Contents**



d) Let N represent the number of pieces of fruit in the basket.
Objective function: $N = x + y$

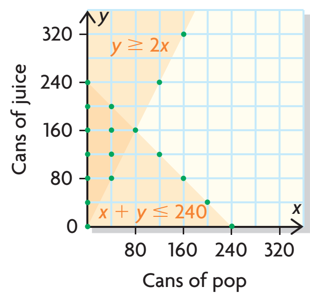
2. Let x represent the number of hamburgers. Let y represent the number of hot dogs. Let R represent the sales revenue.
$\{(x, y) \mid x \leq 150, x \in W, y \in W\}$
$\{(x, y) \mid y \leq 200, x \in W, y \in W\}$
$\{(x, y) \mid x + y \leq 300, x \in W, y \in W\}$
Objective function: $R = 4.75x + 3.25y$

Fast-Food Stand Sales



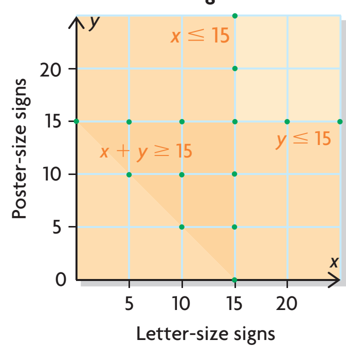
3. Let x represent the number of cans of pop. Let y represent the number of cans of juice. Let R represent the revenue.
$\{(x, y) \mid y \geq 2x, x \in W, y \in W\}$
$\{(x, y) \mid x + y \leq 240, x \in W, y \in W\}$
Objective function: $R = 1.25x + y$

Vending Machine Sales



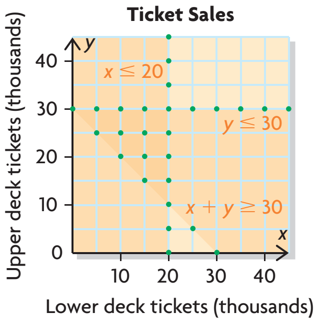
4. Let x represent the number of letter-size signs. Let y represent the number of poster-size signs. Let C represent the cost.
$\{(x, y) \mid x \leq 15, x \in W, y \in W\}$
$\{(x, y) \mid y \leq 15, x \in W, y \in W\}$
$\{(x, y) \mid x + y \geq 15, x \in W, y \in W\}$
Objective function: $C = 9.8x + 15.75y$

Dance Sign Order



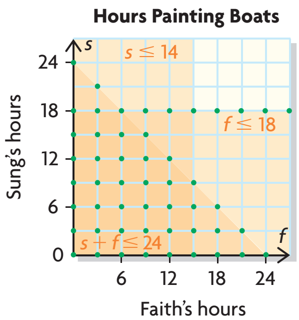
5. Let x represent the number of lower deck tickets. Let y represent the number of upper deck tickets. Let R represent the revenue.
$\{(x, y)| x \leq 20\ 000, x \in \mathrm{W}, y \in \mathrm{W}\}$
$\{(x, y) \mid y \leq 30\ 000, x \in \mathrm{W}, y \in \mathrm{W}\}$
$\{(x, y) \mid x + y \geq 30\ 000, x \in \mathrm{W}, y \in \mathrm{W}\}$

Objective function: $R = 120x + 80y$



6. a) Let s represent the number of hours Sung works. Let f represent the number of hours Faith works. Let B represent the total number of boats painted.
$\{(f, s) \mid s \leq 14, s \in \mathrm{W}, f \in \mathrm{W}\}$
$\{(f, s) \mid f \leq 18, s \in \mathrm{W}, f \in \mathrm{W}\}$
$\{(f, s) \mid s + f \leq 24, s \in \mathrm{W}, f \in \mathrm{W}\}$

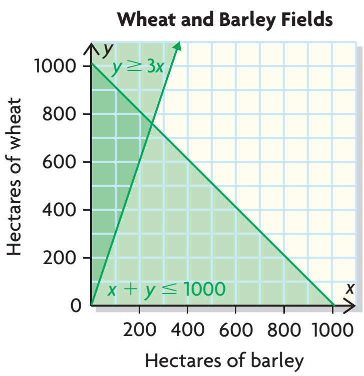
Objective function: $B = \frac{s}{3} + \frac{f}{4}$



b) The new objective function would be $B = \frac{s}{3} + \frac{f}{2}$.

7. Let x represent the number of hectares of barley. Let y represent the number of hectares of wheat. Let R represent the revenue.
$x \geq 0$
$y \geq 0$
$x + y \leq 1000$
$y \geq 3x$

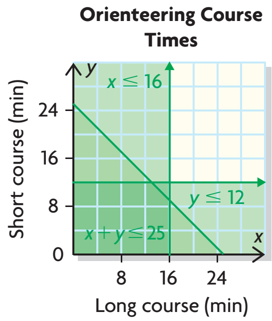
Objective function: $R = (5.25)50x + (3.61)38y$



8. e.g., What is the quantity that must be optimized? What are the quantities that affect the quantity to be optimized? Define these with variables. Are there any restrictions on these variables? What is the system of linear inequalities that describes all the constraints of the problem? What is the objective function?

9. Let x represent time on the long course. Let y represent time on the short course. Let T represent the total time.
$x \geq 0$
$y \geq 0$
$x \leq 16$
$y \leq 12$
$x + y \leq 25$

Objective function: $T = x + y$



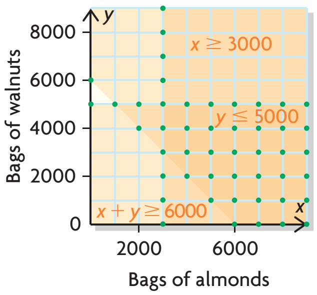
Lesson 6.5, page 334

1. a) Model A: maximum: (11, 3), minimum: near (−4, −12)
   b) Model B: maximum: near (5, 4), minimum: near (0, 9)
2. (−4, 4); e.g., The objective function is the difference of x and y and the other two points have a positive difference.
3. a) (50, 200); e.g., farthest point from both axes
   b) No. These points are not in the feasible region.
   c) (50, 100)
   d) (50, 200); 75 in.
   e) (0, 0) would require no shelving

Lesson 6.6, page 341

1. minimum: (0, 0), maximum: (8, 5)
2. (1.5, 0)
3. a) 12 cars b) 48
4. a) 20 spiders, 25 crickets b) 20 crickets
5. (0, 5)
6. (5, 0)
7. (0, 6)
8. $(h, g) = (3\ 000\ 000, 6\ 000\ 000)$; \$11 850 000
9. a) e.g., Let x represent the number of bags of almonds. Let y represent the number of bags of walnuts.
   i) $\{(x, y) \mid x \geq 3000, x \in \mathrm{W}, y \in \mathrm{W}\}$
   ii) $\{(x, y) \mid y \leq 5000, x \in \mathrm{W}, y \in \mathrm{W}\}$
   iii) $\{(x, y) \mid x + y \geq 6000, x \in \mathrm{W}, y \in \mathrm{W}\}$
   b) The variables must be whole numbers. $x \in \mathrm{W}, y \in \mathrm{W}$

c) Bags of Nuts



d) The feasible region is the set of whole number coordinates that lie on or to the right of the vertical line $x = 3000$, on or below the horizontal line $y = 5000$, and on or above the line $x + y = 6000$.

e) Let C represent the total cost. Let x represent the cost per bag of almonds. Let y represent the cost per bag of walnuts.
$C = 11.19x + 13.1y$

f) 3000 bags of walnuts and 3000 bags of almonds at $72 870.

10. e.g., question 4: minimum cost: 15 letter-size signs; $147.00; question 5: maximum revenue: 20 000 lower and 30 000 upper; $4 800 000

11. 137 economy seats and 8 business seats; maximum revenue: $38 485

12. 1500 min or 25 h; 11 250 min or 187 h 30 min

13. 12 h at $8.75/h and 20 h at $9.00/h; $285

14. 96 small earrings and 24 large earrings; $112 800

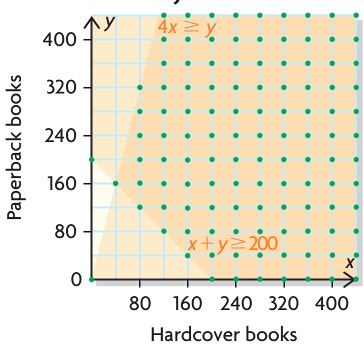
15. 1600 bundles of asphalt shingles and 200 bundles of cedar shakes

16. e.g., What is the graph of the system of linear inequalities? What is the feasible region? What are the vertices of the feasible region? How does the value of the objective function at each vertex compare?

17. e.g., *Problem*: A library is buying both hardcover and paperback books. It plans to purchase at most four times as many paperbacks as hardcover books. Altogether the plan is to purchase no fewer than 200 books. Hardcover books average $35.75 in cost while paperbacks average $12.20. How can the library minimize its costs?
Solution: Let x represent the number of hardcover books. Let y represent the number of paperback books. Let C represent the total cost of the books.
Objective function to minimize: $C = 35.75x + 12.2y$
Constraints and restrictions:
$\{(x, y) \mid x + y \geq 200, x \in W, y \in W\}$
$\{(x, y) \mid 4x \geq y, x \in W, y \in W\}$

The library should purchase 40 hardcover books and 160 paperback books, for a total cost of $3382.00.

Library Book Purchases



Chapter Self-Test, page 347

1. a) The shading for $4 + 2x \leq 3y$ should be above the boundary; the shading for $5x - 2y > 6$ should be to the right of the boundary.
b) The boundary for $x - 3y \leq 4 - 3y$ should be stippled with no solid line. There should be no stippling on the dashed boundary of $2y > x + 7$.

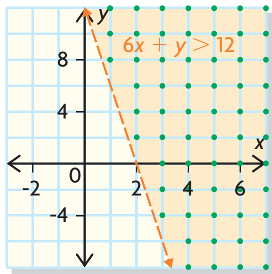
2. minimum: $\left(\frac{-3}{2}, 0\right)$; maximum: (1, 5)

3. 50 ribbon flowers and 0 rosettes; 5 hours

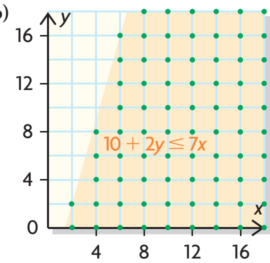
4. 12 vans and 0 minibuses; maximum value: $6600; 120 people

Chapter Review, page 349

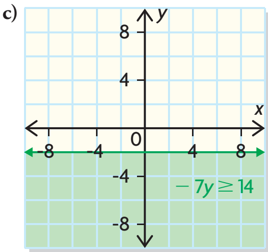
1. a)



b)

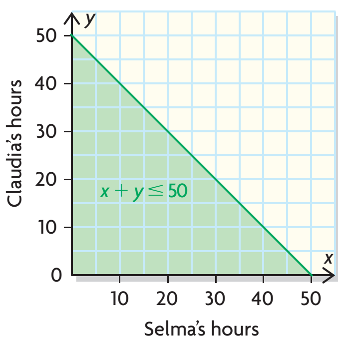


c)



2. a) Let x represent the number of hours Selma works. Let y represent the number of hours Claudia works.
Domain: $x \geq 0, x \in R$
Range: $y \geq 0, y \in R$
$x + y \leq 50$

Hours of Work

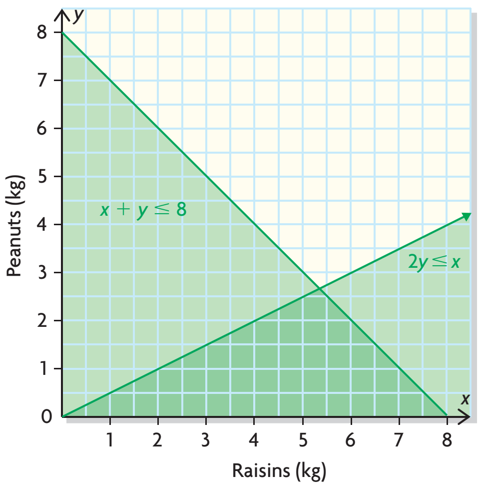


b) e.g., (28, 2), (35, 3), (12, 1.5); by substituting them in the inequality

3. **a)** Let x represent the mass of raisins. Let y represent the mass of peanuts.
Domain: $x \geq 0, x \in R$
Range: $y \geq 0, y \in R$
$2y \leq x$
$x + y \leq 8$

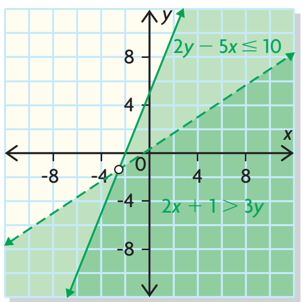
b) e.g., below $2y = x$ and $x + y = 8$

Raisin and Peanut Mixture



c) (6, 2) represents 6 kg of raisins and 2 kg of peanuts;
(4, 1) represents 4 kg of raisins and 1 kg of peanuts; and
(5.5, 1.5) represents 5.5 kg of raisins and 1.5 kg of peanuts.

4. **a)**



b) **i)** The graph would have a stippled boundary (green) in place of the solid one, the solution region of each inequality would be shaded orange, integer points in the system's solution region would be stippled green and the dashed line would be orange.

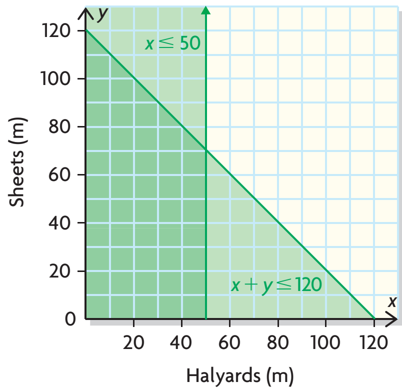
ii) The graph would be shaded orange. The dashed line would be orange, and the graph would be in the first quadrant only. The graph would also show green stippled whole number points in the system's solution region, including points on the x axis. The solid line would be stippled green.

iii) The regions on the opposite sides of the boundaries would be shaded.

5. **a)** The solution belongs to the set of positive real numbers, because you cannot have negative lengths of rope and measurements are continuous.

b) e.g., 45 m halyards and 65 m sheets, or 20 m halyards and 50 m sheets

Sailboat Rope Purchase



6. **a)** $y = 2x - 3$, $y = -2x + 7$

b) $y + 2x < 7$
$y - 2x \geq -3$

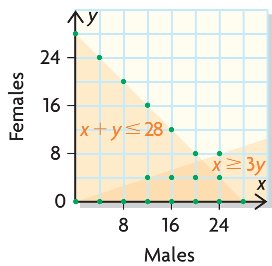
c) e.g., I picked two points on each line to verify. I verified that (2.5, 2), where the boundaries intersect on the graph, satisfied the equations for both boundaries. I picked a point in each solution region, which I could verify in my inequalities, to make sure I correctly interpreted the graph.

d) There are no restrictions on the variables because the solution region is not stippled and it is in all four quadrants.

7. Let x represent the number of male birds sold. Let y represent the number of female birds sold. Let R be the revenue.
$\{(x, y) \mid x + y \leq 28, x \in W, y \in W\}$
$\{(x, y) \mid 3y \leq x, x \in W, y \in W\}$

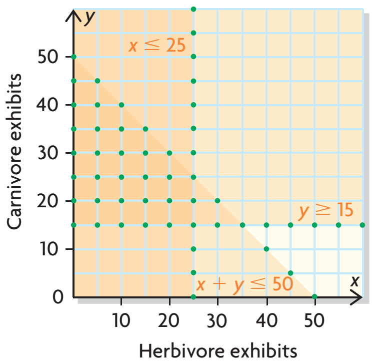
Objective function: $R = 115x + 90y$
28 male birds and 0 female birds

Pet Store Bird Sales



8. Let x represent the number of herbivore exhibits. Let y represent the number of carnivore exhibits. Let R represent the revenue.
$\{(x, y) \mid x \leq 25, x \in W, y \in W\}$
$\{(x, y) \mid y \geq 15, x \in W, y \in W\}$
$\{(x, y) \mid x + y \leq 50, x \in W, y \in W\}$

Objective function: $R = 15x + 18y$
0 herbivore exhibits and 50 carnivore exhibits



9. minimum: (3, 0); maximum: (0, 6)
10. The maximum solution of 4.5 occurs at (3, 0).
11. 72 women's appointments and 18 men's appointments; minimum: 99 h

Chapter 7

Lesson 7.1, page 360

1. **a)** not a quadratic relation **b)** not a quadratic relation **c)** not a quadratic relation **d)** quadratic relation **e)** quadratic relation **f)** not a quadratic relation
2. **a)** not a quadratic relation **b)** quadratic relation **c)** quadratic relation **d)** quadratic relation **e)** not a quadratic relation **f)** not a quadratic relation
3. **b)** 0 **c)** 17 **d)** -6
4. e.g., If $a = 0$, then $y = bx + c$, which is a linear relation, not a quadratic relation.
5. **a)** up, $a > 0$ **b)** down, $a < 0$ **c)** up, $a > 0$ **d)** down, $a < 0$
6. **a)** up **b)** down **c)** up **d)** down

Lesson 7.2, page 368

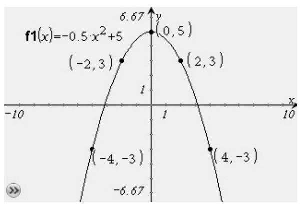
1. **a)** $x = 4$ **b)** $(4, -16)$ **c)** $\{(x, y) \mid x \in R, y \geq -16, y \in R\}$
2. **a)** (0, 8); e.g., (1, 18), $(-1, 2)$ **b)** (0, 0); e.g., (1, 3), $(-1, -5)$
3. **a)** (0, 0), (2, 0); (0, 0); $x = 1$; $(1, -2)$; $\{(x, y) \mid x \in R, y \geq -2, y \in R\}$
b) $(-1, 0)$, (6, 0); (0, 4.5); $x = 2.5$; (2.5, 9.2); $\{(x, y) \mid x \in R, y \leq 9.2, y \in R\}$
4. **a)** $x = 2$; $(2, -1)$; $\{(x, y) \mid x \in R, y \geq -1, y \in R\}$
b) $x = 4$; (4, 28); $\{(x, y) \mid x \in R, y \leq 28, y \in R\}$
c) $x = 3$; $(3, -1)$; $\{(x, y) \mid x \in R, y \leq -1, y \in R\}$
d) $x = 2.5$; $(2.5, -12.25)$; $\{(x, y) \mid x \in R, y \geq -12.25, y \in R\}$
5. **a)** graph d; $(2.5, -12.25)$ **b)** graph b; (4, 28) **c)** graph c; $(3, -1)$ **d)** graph a; $(2, -1)$
6. **a)** maximum of 4 **b)** minimum of -3 **c)** maximum of 2
7. **a) i)**

x	−4	−2	0	2	4
y	−3	3	5	3	−3

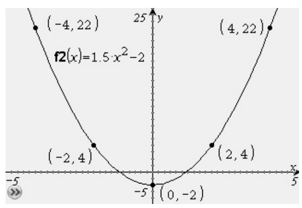
ii)

x	−4	−2	0	2	4
y	22	4	−2	4	22

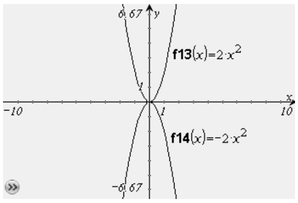
b) i)



ii)



c) i) $\{(x, y) \mid x \in R, y \leq 5, y \in R\}$
ii) $\{(x, y) \mid x \in R, y \geq -2, y \in R\}$

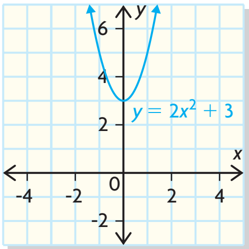
8. a) 

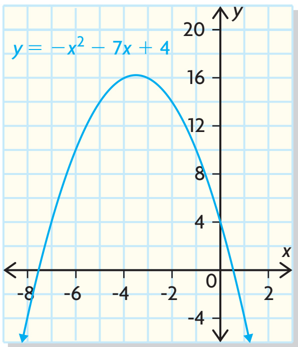
b) e.g., same vertex, axis of symmetry, and shape. One opens up, the other opens down

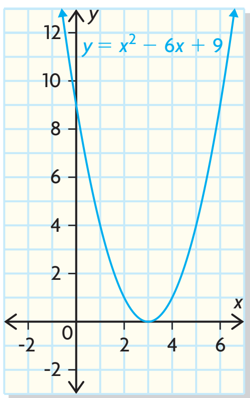
c) e.g., vertex for both is (0, 4), original vertex moves up 4 units for each

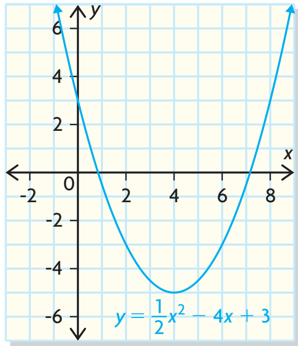
9. a) $x = 3$ c) $x = -2$
b) $x = 5$ d) $x = -1$

10. $x = -3$

11. a) i) 

ii) 

iii) 

iv) 

b) i) $x = 0$; $(0, 3)$
ii) $x = -3.5$; $(-3.5, 16.25)$
iii) $x = 3$; $(3, 0)$
iv) $x = 4$; $(4, -5)$

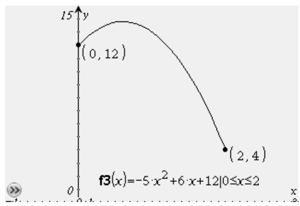
c) i) $\{(x, y) \mid x \in R, y \geq 3, y \in R\}$
ii) $\{(x, y) \mid x \in R, y \leq 16.25, y \in R\}$
iii) $\{(x, y) \mid x \in R, y \geq 0, y \in R\}$
iv) $\{(x, y) \mid x \in R, y \geq -5, y \in R\}$

12. 1.56 seconds

13. a) 31.9 m
b) $\{(x, y) \mid 0 \leq x \leq 5.1, x \in R, 0 \leq y \leq 31.9, y \in R\}$
c) 5.1 seconds

14. $\{(t, h) \mid 0 \leq t \leq 16.3, t \in R, 0 \leq h \leq 326.5, h \in R\}$

15. $\{(x, f(x)) \mid 0 \leq x \leq 2, x \in R, 4 \leq f(x) \leq 13.8, f(x) \in R\}$



16. a) minimum since $a > 0$

b) Method 1: Determine the equation of the axis of symmetry.

$x = \frac{-1 + 5}{2}$

$x = 2$

Determine the y-coordinate of the vertex.

$y = 4(2)^2 - 16(2) + 21$

$y = 16 - 32 + 21$

$y = 5$

The vertex is (2, 5).

Method 2: Create a table of values.

x	−2	−1	0	1	2	3
y	69	41	21	9	5	9

The vertex is halfway between (1, 9) and (3, 9), which have the same y-value, so the vertex is (2, 5).

17. a) The y-coordinates are equal.

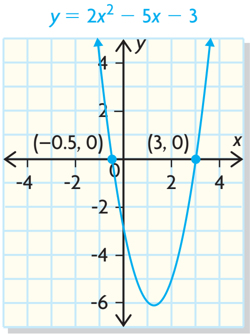
b) e.g., Substitute the x-coordinate from the axis of symmetry into the quadratic equation.

18. e.g., Yes, Gamez Inc.'s profit increased, unless the number of games sold was 900 000; then the profit is the same. For all points except $x = 9$, the second profit function yields a greater profit.

19. $y = -\frac{1}{2}x^2 - 3x + 10$

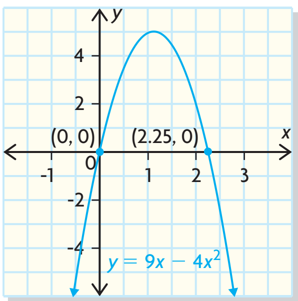
Lesson 7.3, page 379

1. a)



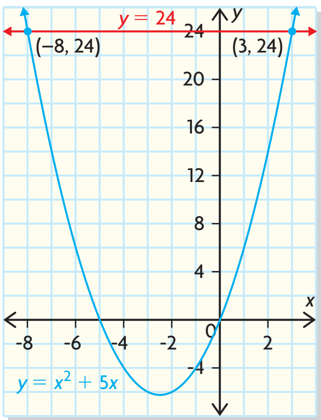
$x = -0.5, 3$

b)



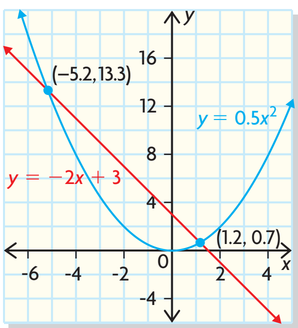
$x = 0, 2.25$

2. a)



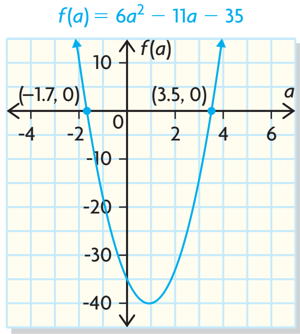
$x = -8, 3$

b)



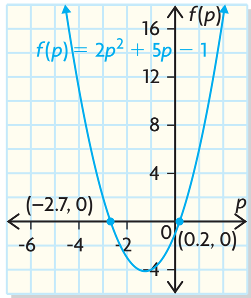
$x \doteq -5.2, 1.2$

3. a) $6a^2 - 11a - 35 = 0$



$a \doteq -1.7, 3.5$

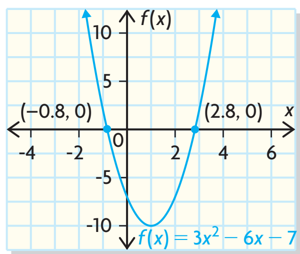
b) $2p^2 + 5p - 1 = 0$



$p \doteq -2.7, 0.2$

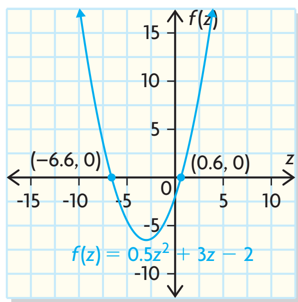
4. a) $x = -2, 5$ **b)** $x = -3$

5. a)



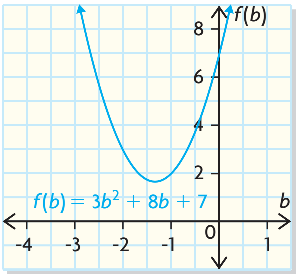
$x \doteq -0.8, 2.8$

b)



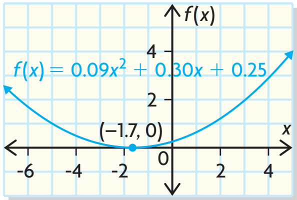
$z \doteq -6.6, 0.6$

c)



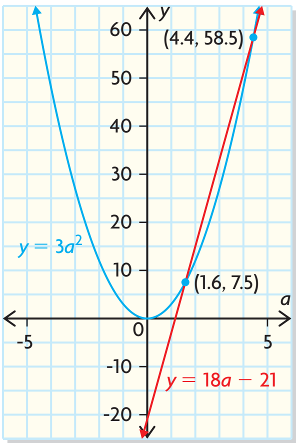
no real roots

d)



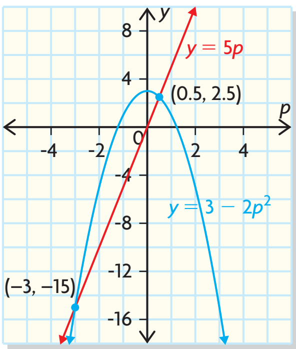
$x \doteq -1.7$

6. a)



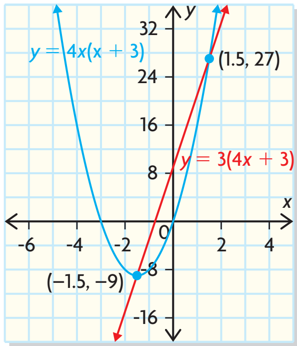
$a \doteq 1.6, 4.4$

b)



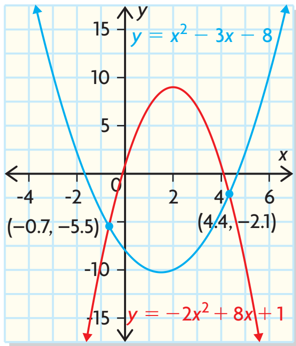
$p = -3, 0.5$

c)



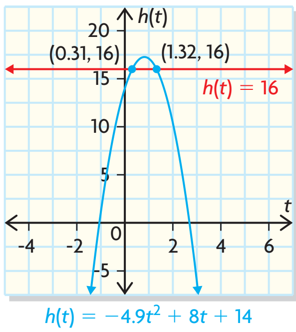
$x = 1.5, -1.5$

d)



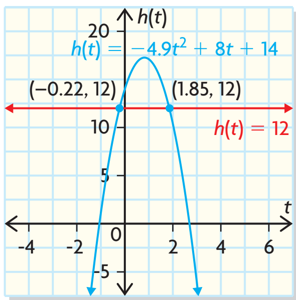
$x \doteq -0.7, 4.4$

7. a) $16 = -4.9t^2 + 8t + 14$



$t = 0.31$ s and $t = 1.32$ s

b) $12 = -4.9t^2 + 8t + 14$



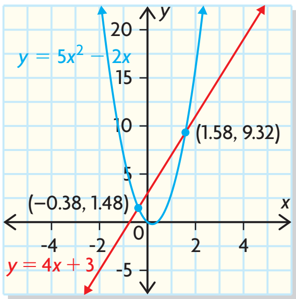
$t \doteq -0.22, 1.85$

$t \geq 0$, $t = 1.85$ s

c) No; the maximum height is less than 18.

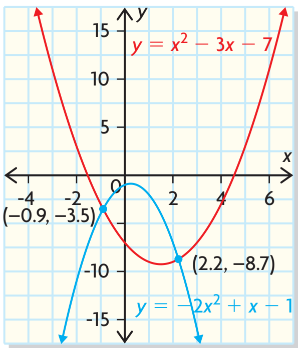
d) $t \doteq 2.69$ s

8. **a)**



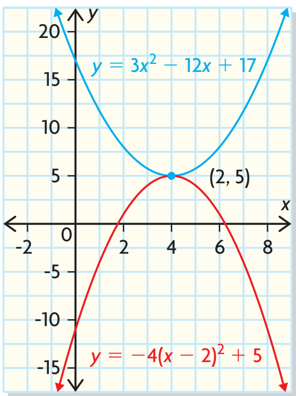
$x = -0.38, 1.58$

b)



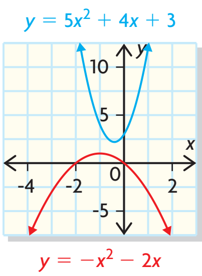
$x \doteq -0.9, 2.2$

c)



$x = 2$

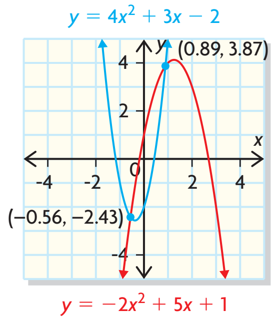
d)



no solution

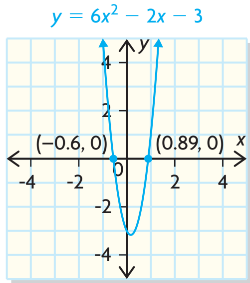
9. Yes, solving $120 = 0.0059s^2 + 0.187s$ indicates that the driver was travelling 127.65 km/h.

10. **a)**



$(-0.56, -2.43)$, $(0.89, 3.87)$

b) $6x^2 - 2x - 3 = 0$



$x = -0.6, 0.89$

c) e.g., I prefer using the method in part b) because there is only one function to graph.

11. 9 m by 13 m

12. **a)** Kevin did not determine the values at the point of intersection, but determined the zeros for the LS function.

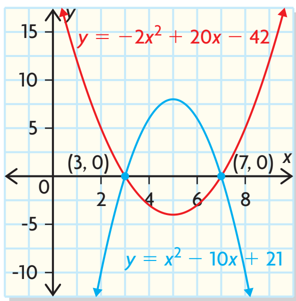
b) $x = 0.153, 2.181$

13. **a)** $x = -23.887, 29.807$

b) $x = -0.605, 7.631$

14. e.g., If the function crosses the x-axis at more than one place, there are two roots; if the function touches the x-axis at one place, there are two equal roots; if the function does not cross the x-axis, there are no real roots.

15.



e.g., $3x^2 - 30x + 63 = 0$, $3x^2 - 30x = -63$, $x^2 - 10x + 21 = 0$

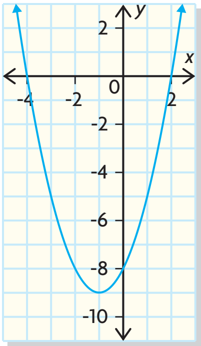
Answers

Lesson 7.4, page 391

1. **a)** iii **b)** ii **c)** v **d)** vi **e)** iv **f)** i

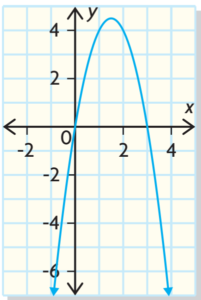
2. **a)** **i)** $x = -4, x = 2$ **ii)** $y = -8$ **iii)** $x = -1$ **iv)** $(-1, -9)$

v) $y = (x + 4)(x - 2)$



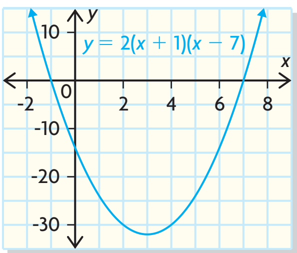
b) **i)** $x = 0, x = 3$ **ii)** $y = 0$ **iii)** $x = 1.5$ **iv)** $(1.5, 4.5)$

v) $y = -2x(x - 3)$



c) **i)** $x = -1, x = 7$ **ii)** $y = -14$ **iii)** $x = 3$ **iv)** $(3, -32)$

v)



3. $y = (x + 2)(x - 4)$

4. **a)** x-intercepts: -1, 1; y-intercept: -1; vertex: $(0, -1)$
equation of the axis of symmetry: $x = 0$

b) x-intercept: -2; y-intercept: 4; vertex: $(-2, 0)$
equation of the axis of symmetry: $x = -2$

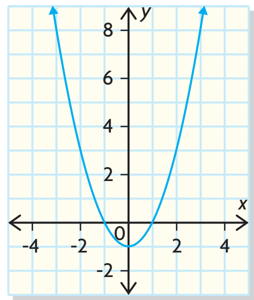
c) x-intercept: 3; y-intercept: 9; vertex: $(3, 0)$
equation of the axis of symmetry: $x = 3$

d) x-intercepts: -1, 2; y-intercept: 4; vertex: $(0.5, 4.5)$
equation of the axis of symmetry: $x = 0.5$

e) x-intercept: 2; y-intercept: 12; vertex: $(2, 0)$
equation of the axis of symmetry: $x = 2$

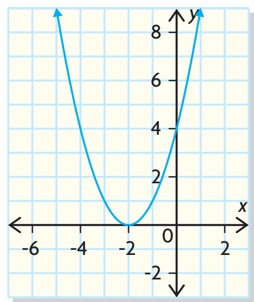
f) x-intercept: 1; y-intercept: 4; vertex: $(1, 0)$
equation of the axis of symmetry: $x = 1$

5. **a)** $y = (x - 1)(x + 1)$



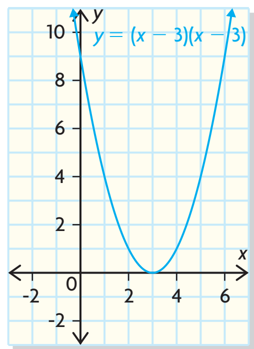
$\{(x, y) \mid x \in \mathrm{R}, y \geq -1, y \in \mathrm{R}\}$

b) $y = (x + 2)(x + 2)$



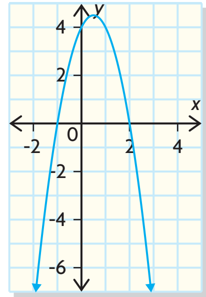
$\{(x, y) \mid x \in \mathrm{R}, y \geq 0, y \in \mathrm{R}\}$

c)



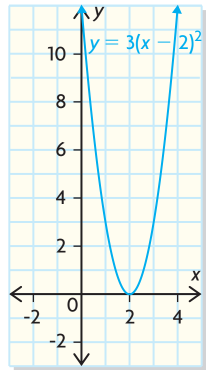
$\{(x, y) \mid x \in \mathrm{R}, y \geq 0, y \in \mathrm{R}\}$

d) $y = -2(x - 2)(x + 1)$



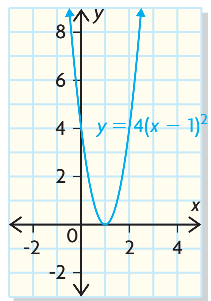
$\{(x, y) \mid x \in \mathrm{R}, y \leq 4.5, y \in \mathrm{R}\}$

e)



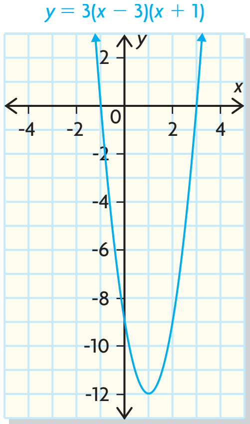
$\{(x, y) \mid x \in \mathrm{R}, y \geq 0, y \in \mathrm{R}\}$

f)



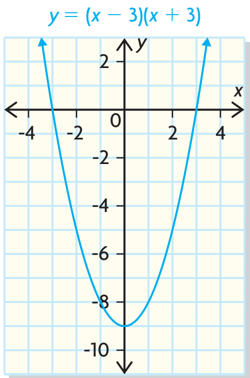
$\{(x, y) \mid x \in \mathrm{R}, y \geq 0, y \in \mathrm{R}\}$

6.



e.g., If $a = 1$ or $a = 2$, the graph would be stretched vertically. If $a = 0$, the graph would be linear. If $a = -1$ or $a = -2$, the graph would be stretched vertically and reflected in the x–axis. If $a = -3$, the graph would be reflected in the x–axis.

7.



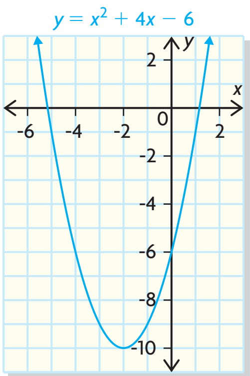
e.g.,
If $s = 2$, zeros at $x = 3$ and $x = -2$, the vertex moves to $(0.5, -6.25)$.
If $s = 1$, zeros at $x = 3$ and $x = -1$, the vertex moves to $(1, -4)$.
If $s = 0$, zeros at $x = 3$ and $x = 0$, the vertex moves to $(1.5, -2.25)$.
If $s = -1$, zeros at $x = 3$ and $x = 1$, the vertex moves to $(2, -1)$.
If $s = -2$, zeros at $x = 3$ and $x = 2$, the vertex moves to $(2.5, -0.25)$.
If $s = -3.8$, zeros at $x = 3$ and $x = 3.8$, the vertex moves to $(3.4, -0.16)$.

8. **a)** 312.5 m² **b)** $\{(x, y) \mid 0 \leq x \leq 25, x \in R, 0 \leq y \leq 312.5, y \in R\}$

9. $12, $720

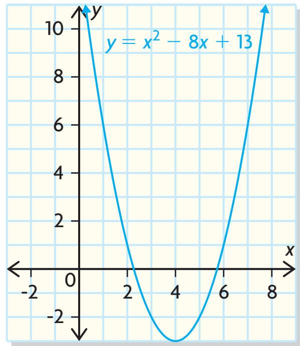
10. **a)** **i)** e.g., $(0, -6)$, $(-4, -6)$ **ii)** $(-2, -10)$

iii)



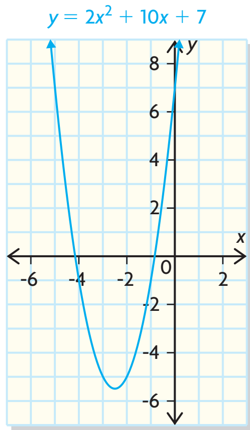
b) **i)** e.g., $(0, 13)$, $(8, 13)$ **ii)** $(4, -3)$

iii)



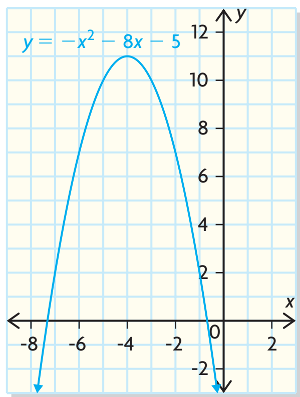
c) **i)** e.g., $(-5, 7)$, $(0, 7)$ **ii)** $(-2.5, -5.5)$

iii)



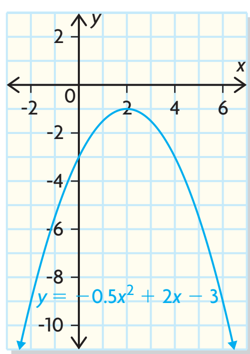
d) **i)** e.g., $(0, -5)$, $(-8, -5)$ **ii)** $(-4, 11)$

iii)



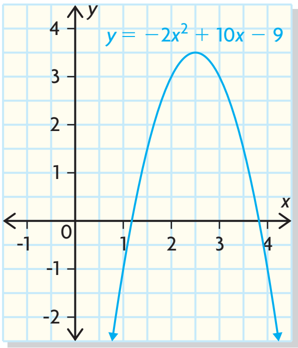
e) **i)** e.g., $(0, -3)$, $(4, -3)$ **ii)** $(2, -1)$

iii)



f) i) e.g., $(0, -9)$, $(5, -9)$ **ii)** $(2.5, 3.5)$

iii)



11. **a)** $y = \frac{1}{2}x^2 - 2x - 6$ **b)** $y = x^2 - 5x + 4$ **c)** $y = -\frac{1}{4}x^2 - x + 3$ **d)** $y = -x^2 + 6x$

12. **a)** e.g., Method 1: Use partial factoring.

$f(x) = -2x^2 + 16x - 24$
$f(x) = -2x(x - 8) - 24$

$-2x = 0$ $\quad$ $x - 8 = 0$
$x = 0$ $\quad$ $x = 8$
$f(0) = -24$ $\quad$ $f(8) = -24$

The points $(0, -24)$ and $(8, -24)$ are the same distance from the axis of symmetry.

$x = \frac{0 + 8}{2}$
$x = 4$

The equation of the axis of symmetry is $x = 4$.

$f(4) = -2(4)^2 + 16(4) - 24$
$f(4) = 8$

The vertex is $(4, 8)$.

Method 2: Factor the equation to determine the x-intercepts.

$f(x) = -2x^2 + 16x - 24$
$f(x) = -2(x^2 - 8x + 12)$
$f(x) = -2(x - 6)(x - 2)$

$x - 6 = 0$ $\quad$ $x - 2 = 0$
$x = 6$ $\quad$ $x = 2$

The x-intercepts are $x = 2$ and $x = 6$.

$x = \frac{2 + 6}{2}$
$x = 4$

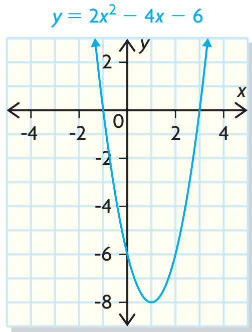
The equation of the axis of symmetry is $x = 4$.

$f(4) = -2(4)^2 + 16(4) - 24$
$f(4) = 8$

The vertex is $(4, 8)$.

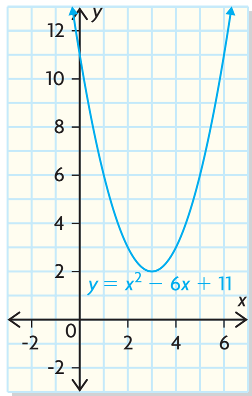
b) e.g., I prefer partial factoring because it is easier to determine the factors.

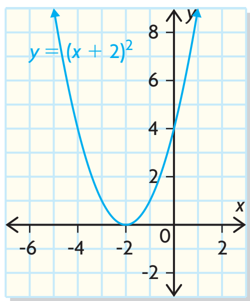
13. $y = 2x^2 - 4x - 6$

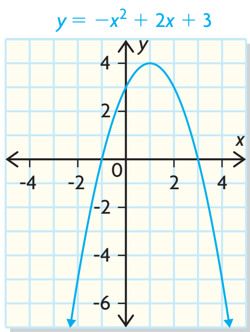


14. A quadratic function can have no zeros, one zero, or two zeros.

e.g.,







15. **a)** $\{(x, y) \mid 0 \le x \le 4, x \in \mathrm{R}, -1 \le y \le 0, y \in \mathrm{R}\}$ **b)** $y = \frac{1}{4}x^2 - x$

16. 12.5 feet by 25 feet

17. **a)** $h = -5.5t^2 + 33t$

b) $\{(t, h \mid 0 \le t \le 6, t \in \mathrm{R}, 0 \le h \le 49.5, h \in \mathrm{R}\}$

18. e.g., The x-intercepts are $x = -3$ and $x = 1$. Therefore, $y = a(x - 1)(x + 3)$. Substitute a point on the graph, say (3, 6), into the equation to obtain $a = \frac{1}{2}$.

19. **a)** $y = -0.019x^2 + 33$

b) $\{(x, y) \mid -33 \le x \le 33, x \in \mathrm{R}, 12 \le y \le 33, y \in \mathrm{R}\}$

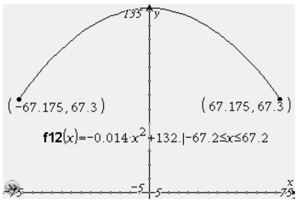
c) $(-41.7, 0), (41.7, 0)$

20. **a)** $y = -0.0144x^2 + 132.279$

b) $\{(x, y) \mid -67.175 \le x \le 67.175, x \in \mathrm{R}, 0 \le y \le 132.279, y \in \mathrm{R}\}$

The grass closest to first and third base is the largest distance to the left or right. The grass closest to second base is the largest vertical distance.

c)



21. 50 feet by 94 feet

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1. **a)** not a quadratic function

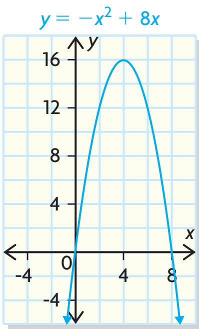
b) quadratic function

c) quadratic function

d) not a quadratic function

2. **a)** $y = 0$

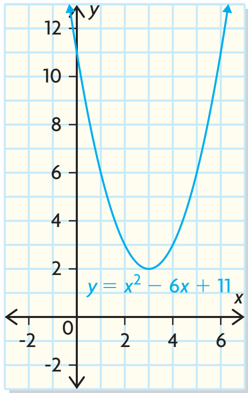
b)



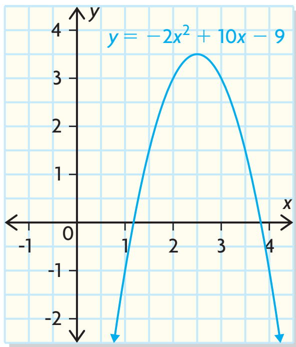
c) $x = 4$; $(4, 16)$; $x = 0, x = 8$; $\{(x, y) \mid x \in \mathrm{R}, y \le 16, y \in \mathrm{R}\}$

3. **a)** e.g., If $a > 0$, then the parabola opens up; if $a < 0$, then the parabola opens down.

b) $a > 0$



$a < 0$



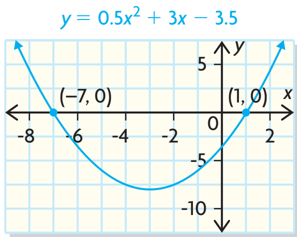
4. **a)** $t = 0, t = 24$

b) 12 seconds, 720 metres

c) 675 metres

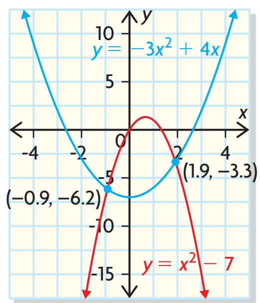
d) $\{(t, h) \mid 0 \le t \le 24, t \in \mathrm{R}, 0 \le h \le 720, h \in \mathrm{R}\}$

5.



$x = -7, 1$

6.



$x = -0.9, 1.9$

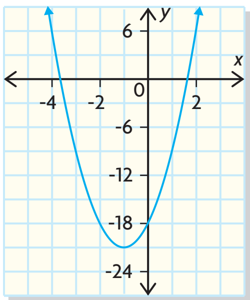
7. 14 s

8. $x = -1$

9. $y = \frac{1}{4}x^2 + \frac{1}{2}x - 2$

10. a) $y = \frac{1}{2}x^2 - 3x - 36$
    b) $(3, -40.5)$
    c) $\{(x, y) \mid x \in R, y \geq -40.5, y \in R\}$

11.



12. $26.25

Lesson 7.5, page 405

1. a) $x = 4, 7$
   b) $x = -3, 10$
   c) $y = -5, -0.5$
   d) $t = -3, 1.25$

2. a) $x = 11, -11$
   b) $r = \frac{10}{3}, -\frac{10}{3}$
   c) $x = 0, 15$
   d) $y = -16, 0$
   e) $s = 6$
   f) $p = -0.25$
   g) $z = 0, 2.5$
   h) $q = 0, \frac{9}{5}$

3. a) $x = -5, 14$
   b) $x = -16, -3$
   c) $a = -4, \frac{1}{3}$
   d) $t = -\frac{4}{3}, \frac{5}{2}$

4. a) $x = -4, \frac{3}{2}$
   b) $x = -3, \frac{3}{4}$
   c) $d = -\frac{3}{7}$
   d) $g = \frac{13}{9}, -\frac{13}{9}$

5. a) $x = -0.75, 1.8$
   b) e.g., Geeta may have had the wrong signs between the terms within each factor.

6. a) $u = -7, 9$
   b) $x = -4, -2$
   c) $y = -6, 2$
   d) $k = -5$

7. e.g., $x^2 + 17x + 60 = 0$

8. The price of the ticket should be either $1.50 or $3.50. (That is, $x = -5$ or $x = 15$.)

9. $x = \frac{1}{4}, \frac{8}{5}$

10. Going from the second line to the third line, 100 divided by 5 is 20, not 25. Also, in the final step, it is possible that the final result could be positive or negative. Therefore, the two solutions are $a = -\sqrt{20}$ or $a = \sqrt{20}$.

11. The first line was incorrectly factored:
$4r^2 - 9r = 0$
$r(4r - 9) = 0$
$r = 0$ or $4r - 9 = 0$
$4r = 9$
$r = \frac{9}{4}$
$r = 0$ or $r = \frac{9}{4}$

12. a) e.g., $y = 8x^2 + 2x - 3$
    b) e.g., No, we had different functions.
    c) e.g., $y = 16x^2 + 4x - 6$, $y = -24x^2 - 6x + 9$

13. a) She must sell either 600 or 1800 posters to break even.
    b) She must sell either 800 or 1600 posters to earn a profit of $5000.
    c) She must sell 1200 posters to earn $9000.
    d) D: $n \geq 0$; R: $-27 \leq P \leq 9$
    If Sanela sells 0 posters, she will incur a loss of $27 000; her maximum profit is $9000.

14. a) 7
    b) D: $0 \leq t \leq 7$, where t is the time in seconds, since the rock begins to fall at 0 s and hits the water at 7 s

15. a) e.g., $x^2 - 2x - 8 = 0$
    b) e.g., $x^2 - 2x + 9 = 0$; I changed the "c" term.

16. a) i) Write the equation in standard form.
    ii) Factor fully.
    iii) Set each factor with a variable equal to zero (since the product is zero, one factor must be equal to zero).
    iv) Solve.
    b) When the quadratic equation is factorable, solve by factoring; otherwise, solve by graphing.

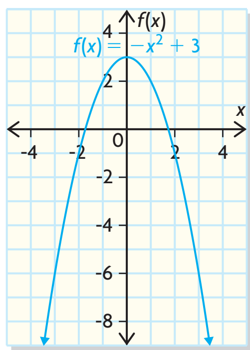
17. a) Since the equation is factorable, I can predict that it is a difference of squares.
    b) $x = -6$
    c) $\left(\sqrt{ax} + \sqrt{c}\right)\left(\sqrt{ax} - \sqrt{c}\right) = 0$, where $\frac{\sqrt{c}}{\sqrt{a}} = 6$
    d) $ax^2 - c = 0$

18. 10 cm, 24 cm, and 26 cm

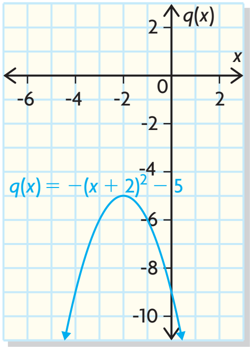
Lesson 7.6, page 417

1. a) i) upward ii) $(3, 7)$ iii) $x = 3$
   b) i) downward ii) $(-7, -3)$ iii) $x = -7$
   c) i) upward ii) $(2, -9)$ iii) $x = 2$
   d) i) upward ii) $(-1, 10)$ iii) $x = -1$
   e) i) downward ii) $(0, 5)$ iii) $x = 0$

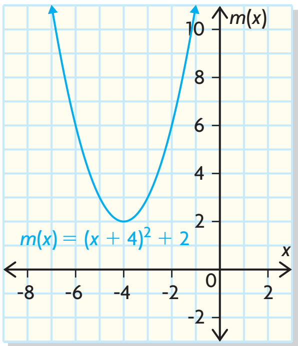
2. a) maximum, 2 x-intercepts



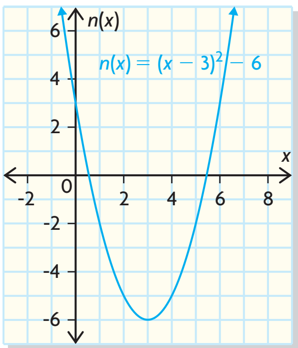
b) maximum, 0 x-intercepts



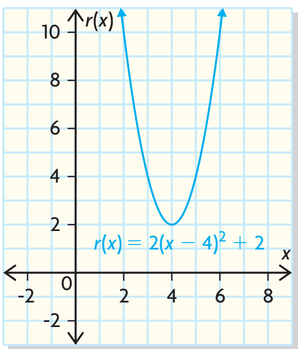
c) minimum, 0 x-intercepts



d) minimum, 2 x-intercepts



e) minimum, 0 x-intercepts



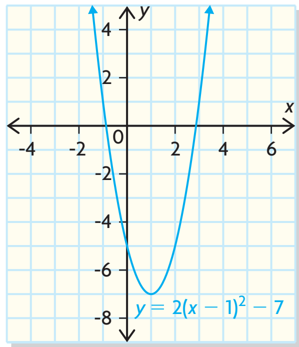
3. -3

4. C. The vertex is (3, 5) and passes through the point $(0, -1)$.

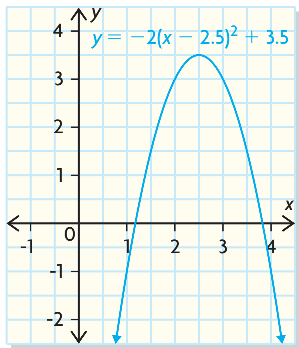
5. **a)** iv; The vertex is (3, 0). **c)** i; The vertex is $(0, -3)$.
b) iii; The vertex is $(-4, -2)$. **d)** ii; The vertex is (4, 2).

6. If $a > 0$, the parabola contains a minimum value. If $a < 0$, the parabola contains a maximum value.

$a > 0, y = 2(x - 1)^2 - 7$



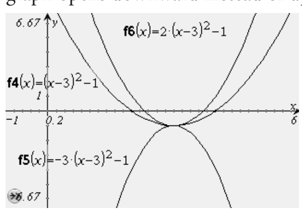
$a < 0, y = -2(x - 2.5)^2 + 3.5$



7. red, $y = (x + 4)^2$; $a = 1$
purple, $y = (x - 4)^2$; $a = 1$
blue, $y = -(x - 4)^2 - 4$; $a = -1$
orange, $y = x^2 + 4$; $a = 1$
green, $y = -x^2 - 4$; $a = -1$
The parabolas are congruent.

8. **a)** $x = 9$ **c)** 6.5 ft
b) 8 ft **d)** $\{h(x) \mid 6.5 \le h \le 8, h \in \mathbb{R}\}$

9. **a)** e.g., $y = (x - 3)^2 - 1$, $y = 2(x - 3)^2 - 1$, $y = -3(x - 3)^2 - 1$
b) The second graph is narrower than the first graph, and the third graph opens downward instead of upward.
c)



e.g., My predictions were accurate.

10. e.g., The vertex is (1, −9), the graph opens upward, the equation of the axis of symmetry is $x = 1$, and the y-intercept is (0, −7). I would draw a parabola that has all of these features.

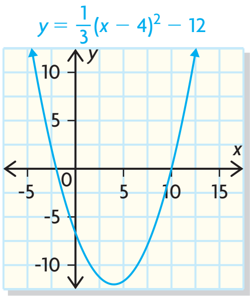
11. **a)** $y = -\frac{1}{4}x^2 + 36$ **b)** $y = -\frac{2}{9}(x - 3)^2 + 2$

12. **a)** $y = a(x - 4)^2 - 12,\ a \neq 0,\ a \in \mathrm{R}$

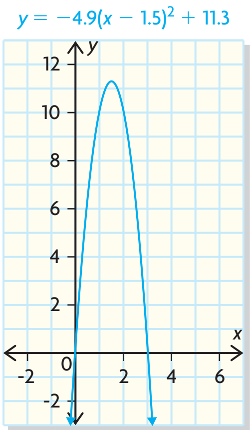
b) $y = \frac{1}{3}(x - 4)^2 - 12$

c) $\{(x, y) \mid x \in \mathrm{R}, y \geq -12, y \in \mathrm{R}\}$

d)



13. **a)** zeros; 0, 3



b) e.g., One zero represents the location of the sprinkler and the other zero represents where the water lands on the grass.

14. **a)** $y = -2.4(x + 3.5)^2 + 15$

b) $\{(x, y) \mid x \in \mathrm{R}, y \leq 15, y \in \mathrm{R}\}$

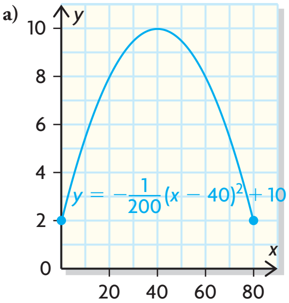
15. **a)** $0.68(x - 2.5)^2 + 0.5$

b) 0.68 m

c) $\{(x, y) \mid 0 \leq x \leq 5, x \in \mathrm{R}, 0.5 \leq y \leq 1, y \in \mathrm{R}\}$

16. e.g., Agree. It is easier to graph the quadratic function when it is in vertex form because you can determine the vertex, the y-intercept, and direction of the graph without doing any calculations.

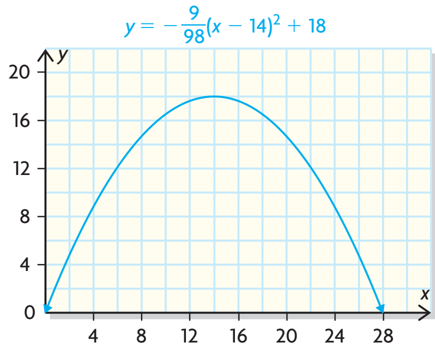
17. **a)**



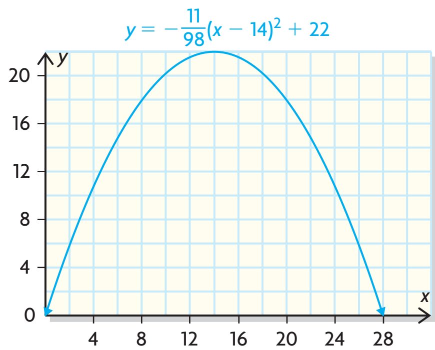
b) The atlatl dart was 20 yd from Peter as it rose in the air, then 60 yd as it came down.

18. **a)** $h(t) = 5(t - 10)^2 + 20$ **b)** 20 m **c)** 20 s

19. e.g., $-\frac{9}{98}(x - 14)^2 + 18$

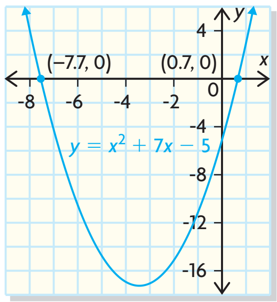


e.g., $-\frac{11}{98}(x - 14)^2 + 22$

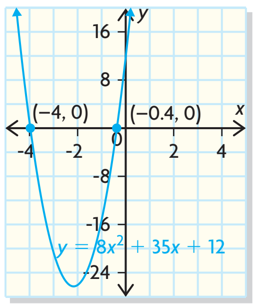


Lesson 7.7, page 427

1. **a)** $x = \frac{-7 - \sqrt{69}}{2}, \frac{-7 + \sqrt{69}}{2}$

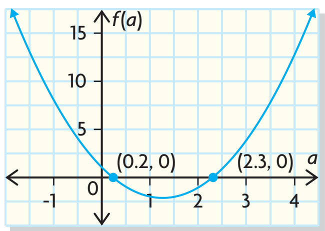


b) $x = -4, -0.375$

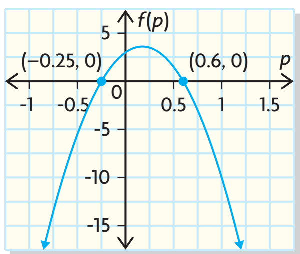


c) $a = \frac{5 - \sqrt{17}}{4}, \frac{5 + \sqrt{17}}{4}$

$f(a) = 2a^2 - 5a + 1$



d) $p = -0.25, 0.6$



$f(p) = -20p^2 + 7p + 3$

2. **a)** $x = -6, 1$

b) $x = -\frac{4}{9}, 0$

c) $x = 2.2, -2.2$

d) $x = -\frac{5}{4}, \frac{8}{3}$

3. e.g., I preferred factoring because it takes less time and there is less room for errors.

4. **a)** $x = \frac{-5 - \sqrt{133}}{6}, \frac{-5 + \sqrt{133}}{6}$

b) $x = \frac{7 - \sqrt{1102}}{39}, \frac{7 + \sqrt{1102}}{39}$

c) $x = \frac{3 - \sqrt{3}}{2}, \frac{3 + \sqrt{3}}{2}$

d) no solution

5. The roots are correct.

6. **a)** $x = \frac{3 - 2\sqrt{3}}{3}, \frac{3 + 2\sqrt{3}}{3}$

b) $x = -4 - \sqrt{13}, -4 + \sqrt{13}$

c) $x = \frac{-2 - \sqrt{6}}{4}, \frac{-2 + \sqrt{6}}{4}$

d) $x = \frac{2 - \sqrt{5}}{3}, \frac{2 + \sqrt{5}}{3}$

7. **a)** \$0.73, \$19.27

b) \$10

8. **a)** 5.5 s

b) e.g., about 10 s as 250 m is twice 125 m

c) 7.6 s

d) e.g., My prediction was not close.

9. **a)** It may be possible, but the factors would not be whole numbers.

b) $z = -0.75$

c) e.g., I used the formula because I find it most efficient.

10. **a)** 7.28 s

b) 1.77 s; The ball would be in flight 5.51 s longer on the Moon.

11. 0.25 m

12. e.g.,

- The quadratic formula can be used to solve any quadratic equation.
- You can use it to solve a factorable equation if you find it too difficult to factor.
- The radicand can be used to tell you about the solution.
  - If it is a perfect square, then the equation is factorable. Both roots are rational numbers.
  - If it is not a perfect square, then the roots can be given as a decimal approximation, or you can choose to leave the radical in the solution and give the exact values.
  - If it is negative, then there is no solution.

13. **a)** $-\frac{b}{a}$

b) $\frac{c}{a}$

c) $x = 0.5, 0.8$; sum $= 1.3$; product $= 0.4$

d) Yes, the answers match.

e) 1. d) sum: $\frac{7}{20}$; product: $-\frac{3}{20}$

2. a) sum: -5; product: -6

5. sum: -2; product: -15

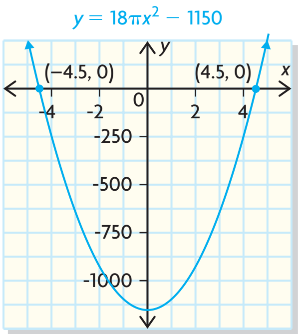
7. sum: 20; product: 14

f) e.g., Determine the sum and product of your proposed solutions, then check to see if they match the results obtained from the formulas in parts a) and b).

Lesson 7.8, page 436

1. **a)** e.g., Graph the equations and determine the intersection.
b) 23.76 m
c) e.g., Factor the equation to determine the *x*-intercepts.

2. **a)** 4.51 cm
b)



c) e.g., I prefer the method in part a) because it takes less time.

3. −8, 19

4. 4.24 cm

5. about 1.57 s

6. 8:27 a.m.

7. **a)** $E(x) = -5x^2 + 75x + 5000$
b) \$40
c) \$32.50

8. −14 and −13, or 13 and 14

9. about 29.0 cm

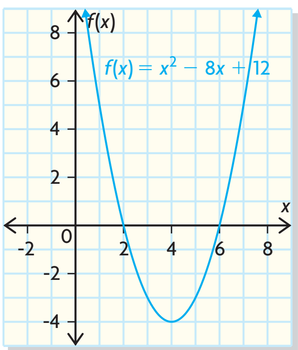
10. e.g., Underline key words, write what is given, write what you need to figure out, draw a picture, use a strategy previously used, and ask yourself if the answer is probable.

11. 6:30 pm

12. 14.4 cm

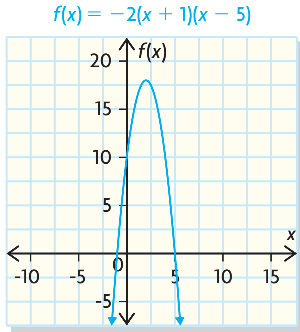
Chapter Self-Test, page 440

1. **a)**



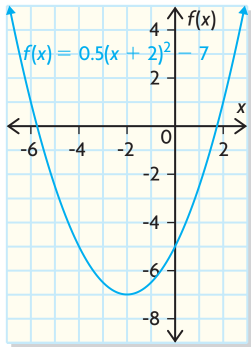
e.g., I used partial factoring to determine two points on the parabola with the same *y*-coordinate, then the axis of symmetry, and then the *y*-coordinate of the vertex.

b)



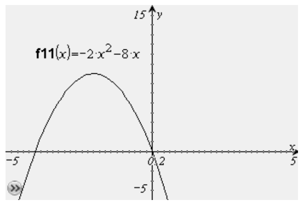
e.g., I plotted the *x*-intercepts and the *y*-intercept.

c)



e.g., I plotted the vertex and the *y*-intercept.

d)



e.g., I factored the equation to determine the vertex and *x*-intercepts.

2. **a)** *x*-intercepts: −3, 5; *y*-intercept: 15; vertex: (1, 16)
equation of the axis of symmetry: $x = 1$
b) *x*-intercepts: 1.5, −4; *y*-intercept: −12; vertex: (−1.25, −15.125)
equation of the axis of symmetry: $x = -1.25$

3. **a)** 10 s **b)** 256 ft **c)** 400 ft

4. $y = \frac{1}{3}x^2 - 2x - 4$

5. \$900

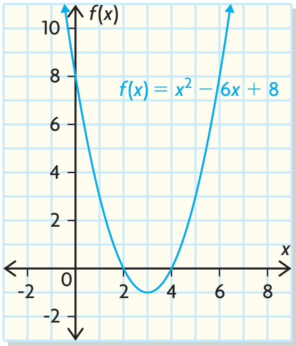
6. **a)** $x = -8, -3$
b) $a = -4, \frac{1}{8}$
c) $c = -1, 6$
d) $x = -\frac{2}{5}, -\frac{1}{4}$

7. **a)** $x \doteq -6.27, 1.27$
b) $x \doteq -0.23, 3.23$
c) $x = \frac{3}{5}$
d) no solution

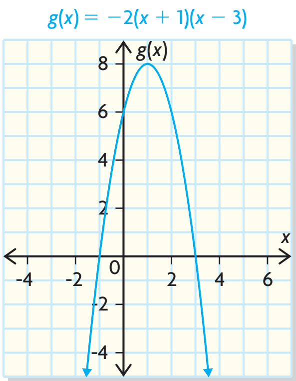
8. **a)** 56.2 m
b) about 4 m

Chapter Review, page 443

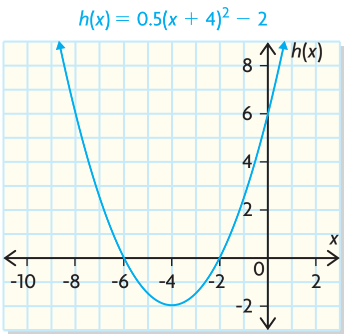
1. a)



b)



c)

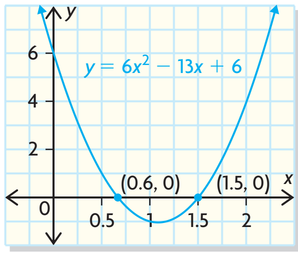


2. (2, 7)

3. a) $y = -\frac{3}{2}(x+1)^2 + 2$

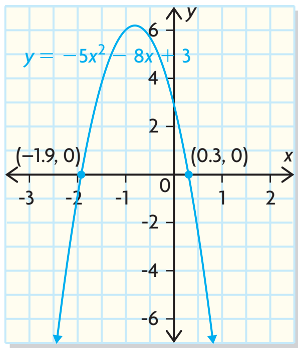
b) $\{(x, y) \mid -1 \leq x \leq 0, x \in \mathbb{R}, 0.5 \leq y \leq 2, y \in \mathbb{R}\}$

4. a)



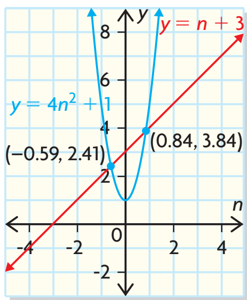
$x \doteq 0.6, 1.5$

b)



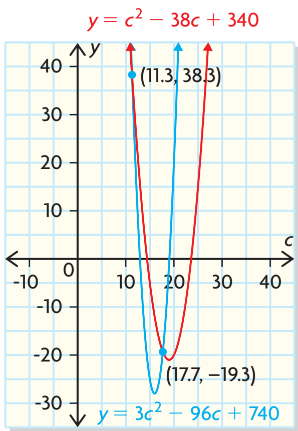
$x \doteq -1.9, 0.3$

c)



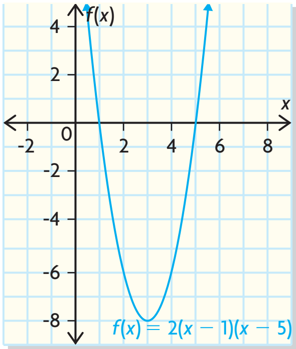
$n \doteq -0.59, 0.84$

d)



$c \doteq 11.3, 17.7$

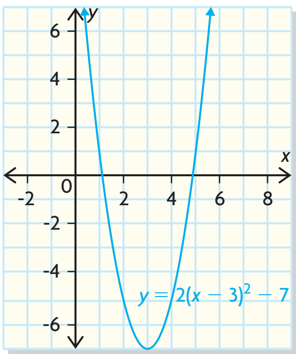
Answers

5. a) $f(x) = 2(x - 1)(x - 5)$
b) zeros: $x = 1, x = 5$
equation of the axis of symmetry: $x = 3$
c) $\{(x, y) \mid x \in R, y \geq -8, y \in R\}$
d)


6. $x = -1.5, 4$

7. a) (1, 2)
b) (−2.5, 0.25)

8. a) $s = -5, 12$
b) $a = -3, -2$
c) $d = 3.25, -3.25$
d) $x = 4.5, -4.5$

9. a) upward
b) $x = 3$, (3, −7)
c) $\{(x, y) \mid x \in R, y \geq -7, y \in R\}$
d)


10. $y = -3(x + 4)(x + 2)$

11. $y = -\frac{1}{4}(x - 3)^2 - 5$

12. $y = -0.45x^2 + 45$

13. a) $x = \frac{11}{13}, \frac{16}{9}$
b) $f \doteq 0.73, -2.73$
c) $h \doteq 0.52, -1.38$
d) no solution

14. The numbers are 1, 3, 5 or 3, 5, 7.

15. The other side is 45 cm; the hypotenuse is 51 cm.

16. e.g., $y = -\frac{1}{360}(x - 65)(x + 65)$, $y = -\frac{1}{360}x^2 + \frac{13}{36}x$

17. 1:59 p.m.

Chapter 8

Lesson 8.1, page 458

1. a) store A: \$8.50/kg; store B: \$7.35/kg; store B has the lower rate
b) station A: \$0.94/L; station B: \$0.98/L; station A has the lower rate

2. a) tank A: 71 L/h; tank B: 69 L/h; tank A has the greater rate
b) person A: 5 m/s; person B: 3 m/s; person A has the greater rate

3. a) 20 s to 28 s; 28 s to 32 s
b) 28 s; 32 s
c) distance does not change; speed is zero

4. a) bottles: \$0.00 175/mL; boxes: \$0.001 66/mL
b) Boxes have the lower unit cost.

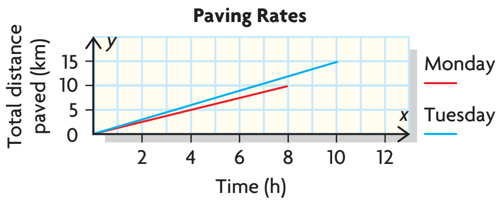
5. 925 mL container: \$0.022/mL; 3.54 L container: \$0.015/mL; The larger container has the lower unit cost.

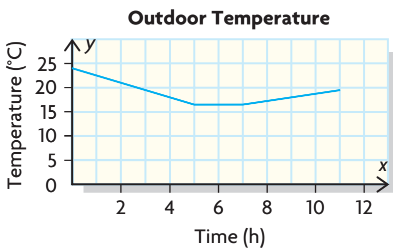
6. aerobics: 7 cal/min; hockey: 8 cal/min; She burns calories at a greater rate playing hockey.

7. a) 10 lb for \$17.40 is the same as \$3.83/kg; \$3.61/kg is the lower rate.
b) 6 mph is the same as 10 km/h; 2 km in 10 min is the same as 12 km/h; the first rate is lower.
c) 35.1 L for 450 km is the same as 7.8 L/100 km; this is the lower rate
d) 30 m/s is the same as 108 km/h; 100 km/h is the lower rate.

8. Farmer's Co-op: \$0.852/lb; pet store: \$0.623/lb; pet store has the lower rate

9. telephone company: \$24/year; Internet: \$28.95/year

10.


11.


12. e.g., In the first 10 min, the shuttle was driven away from the airport to pick up and drop off passengers at three different hotels; the farthest hotel was about 9 km away. Then the shuttle was driven toward the airport for one more pick-up/drop-off about 7.5 km from the airport. It continued toward the airport and stopped at one more hotel, where David disembarked. The whole trip took about 22 min.

13. a) graduated cylinder b) flask c) beaker d) drinking glass

14. The rates for 1990–1995 and 1995–2000 are both 4.8 megatonnes/year.

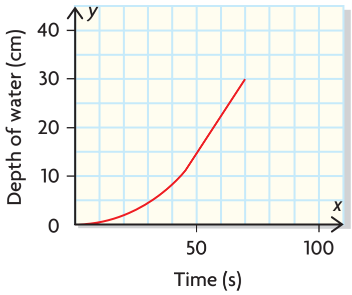
15. The greatest speed difference is in the 700 m to 1100 m segment, by 0.113 m/s.

16. e.g.,

a) An estimate is sufficient when you only need to know which rate is better, such as which car uses less fuel per kilometre. A precise answer is needed if you want to know how much fuel you will save for a particular trip.

b) A graphing strategy is a good approach for comparing rates because you can visually compare the slopes. For example, a steeper slope for one lap of a car race on a graph of distance versus time means a faster speed. A numerical strategy is better if you want to know exactly how much faster one lap was compared to another.

17. **Depth of Water in a Flask**



18. **a)** about 14 300

b) e.g., 27 079 MIPS; about 3693

Lesson 8.2, page 466

1. **a)** 9 L **b)** 3 min **c)** $12.75 **d)** about 30 mL

2. **a)** Supersaver: $0.25/can; Gord: $0.28/can; Supersaver has the lower unit price.

b) e.g., size of container, amount that must be bought

3. 56 turns

4. 18 games

5. 6% per year

6. e.g.,

a) cost for meat in a grocery store

b) amount of medicine per body mass

c) cost for cold cuts at the deli counter

d) change in temperature as altitude changes when climbing a mountain

e) density of a substance

f) cost of flooring at a hardware store

7. 4 min 16 s

8. 44 min

9. 20

10. 867 h

Strategy 1: She works 50 h every 3 weeks; therefore, she works approximately 16.667 h in 1 week (50 h/3 weeks). Since there are 52 weeks in a year, she works approximately 866.7 h in a year (16.667 h/week × 52 weeks/year)

Strategy 2:

$$\frac{50\text{ h}}{3\text{ weeks}} = \frac{x}{52\text{ weeks}}$$

Solve for x to get approximately 866.7 h.

11. **a)** 65.2 km/h **b)** 9.8 L/100 km **c)** $1.09/L

12. $704.50

13. **a)** $817.95

b) no, for 8 weeks she would need 844.8 pounds

c) e.g., What is the food's shelf-life? How much space will be needed to store the food? What are the shipping charges?

14. e.g.,

a) about 400 000 ha

b) about $19.2 million

15. store A: $0.416/L; store B: $0.441/L

Store A, because it is closer and the water is less expensive per litre.

16. 7:24 a.m.

17. −2.2 °C

18. **a)** Bren's Interior Design

b) e.g., The distance to the store or whether a second coat will be needed.

19. 1268

20. 4.7 h

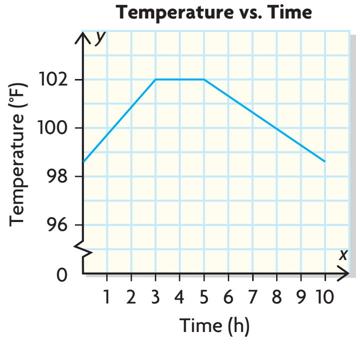
Mid-Chapter Review, page 473

1. Carol is faster because Jed's keying rate is 58 words/min.

2. Stan paid less per litre, because Harry paid $0.985/L.

3. **a)** 4 Cal/min **b)** 30 L/day **c)** $8.40/kg **d)** 5 km in 20 min

4. **a)** **Temperature vs. Time**



b) interval 0 h to 3 h

5. **a)** The interval about 28 s to 35 s; the slope is steepest over this interval.

b) The interval about 35 s to 60 s; the slope is least steep over this interval.

c) about 19 m/s; about 5 m/s

d) 8.3 m/s

6. **a)** U.S. store, $114.47

b) e.g., return/exchange/repair policies, service, custom duties, delivery time, shipping costs

7. 1000 mi

8. $102.20

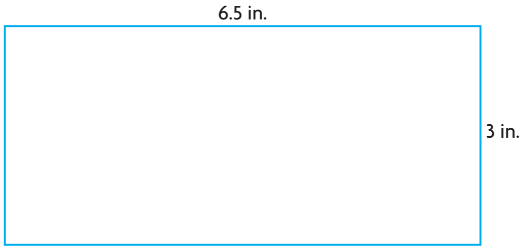
9. **a)** 7.28 m/s, 7.17 m/s

b) The average speed is slightly less for the race that is slightly longer.

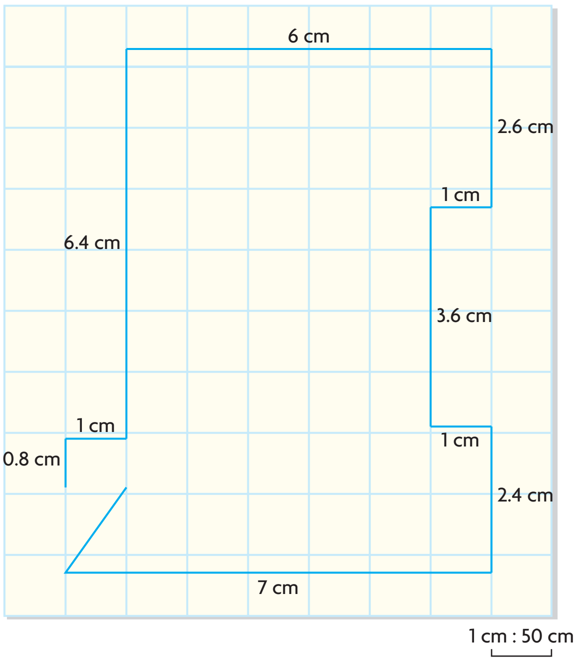
c) e.g., Longer races typically have lower average speeds.

Lesson 8.3, page 479

1. **a)** $\frac{3}{5} = 60\%$ **b)** $\frac{3}{2} = 150\%$
2. **a)** original smaller **b)** original larger **c)** original larger
3. **a)** 5 in. : 6 ft or 5 in. : 72 in. **b)** $\frac{5}{72}$
4. $g = 4.0$ cm, $h \doteq 5.3$ cm, $x = 6.0$ m, $y = 7.5$ m
5. e.g.,
   a) 1.2 **b)** 1.8 **c)** 0.9
6. **a)** 1, 4.0 m by 5.0 m; 2, 4.0 m by 4.0 m; 3, 4.0 m by 4.0 m
   b) 4.8 m by 4.0 m
   c) bedroom 1, 20.0 m^2
7. **a)** e.g., 1 in. : 100 ft
   b) e.g.,

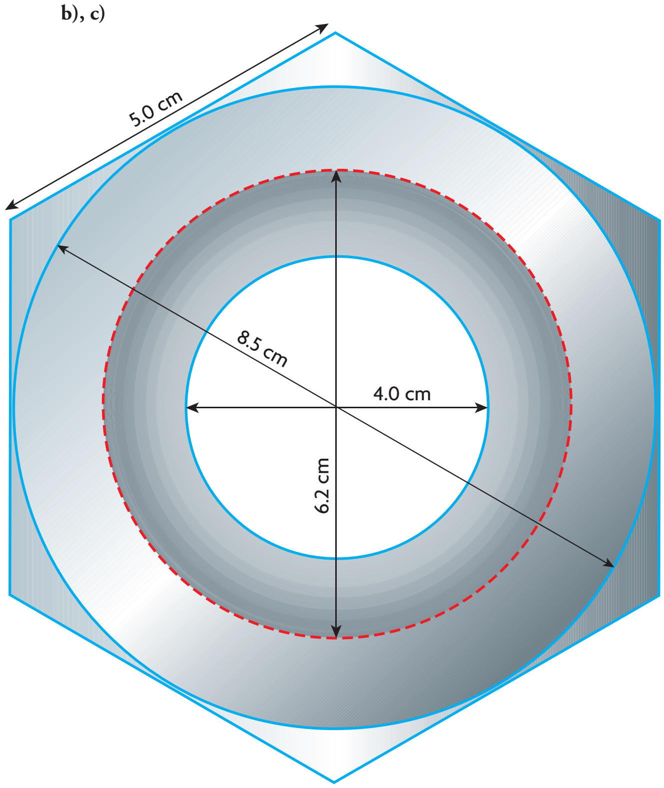


8.

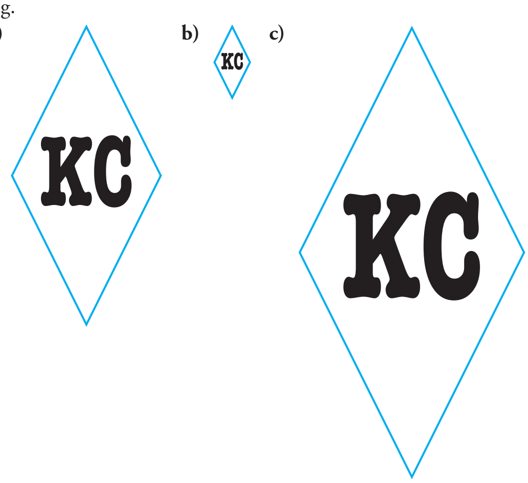


9. The diagram should measure 13.5 cm by 9 cm.

10. e.g.,
   a) diameters: 1.6 cm, 2.5 cm, 3.4 cm; hexagon side: 2.0 cm
   b), c)



11. 0.25 mm
12. **a) i)** about 629 km
   ii) about 557 km
   b) Yellowknife and Fort Providence
13. **a)** 15 m **b)** 11.8 m^2
14. **a)** 3 **b)** $\frac{1}{20}$ **c)** 40 **d)** $\frac{1}{110}$
15. e.g., The diagram could be a rectangle measuring 18 cm by 14 cm
16. **a)** 4 m^2 **b)** 2 m **c)** 5 mm : 2 m **d)** $\frac{1}{400}$
17. width = 36.6 in., height = 20.6 in.
18. e.g.
   a) **b)** **c)**



19. e.g., The dimensions of the space you actually have for your scale diagram; how large you want the scale diagram to be in that space; and a comparison of the ratio of the dimensions of the available space to the ratio of the dimensions of the original.

20. **a)** 0.65 **b)** 7.8 in. by 5.2 in.

Lesson 8.4, page 487

1. **a)** 4 **b)** 12 cm², 192 cm² **c)** 16

2.

Length of Base (cm)	Height of Triangle (cm)	Scale Factor	Area (cm²)	$\frac{\text{Area of scaled triangle}}{\text{Area of original triangle}}$
3.0	4.0	1	6.0	1
9.0	12.0	3	54.0	9
1.5	2.0	0.5	1.5	0.25
30.0	40.0	10	600.0	100
0.75	1.0	25%	0.375	0.0625

3. 1050 cm²

4. **a)** 44 units² **b)** 52 units² **c)** 50 units²

5. **a)** 2.5 units² **b)** 1.3 units²

6. **a)** 6 in. by 9 in.
   b) 225%
   c) e.g., Enlarge each side by 150%, then multiply the new side lengths, or calculate the area of the smaller photo, then multiply by 2.25.

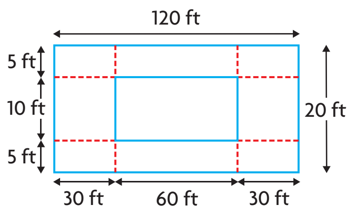
7. Enlarge each side length using a scale factor of 2.

8. **a)** $\frac{2}{3}$ **b)** 64 cm²

9. garage: 600 m², office: 100 m²

10. **a)** $65 000 **b)** $280 000

11.



12. 8 cm² and 32 cm²

13. **a)** 1.5 **b)** 0.5

14. **a)** 4
    b) The perimeter of the large triangle is 4 times the perimeter of the small triangle; the area of the large triangle is 4^2 times the area of the small triangle.

15. **a)** 0.152 m : 7600 m = 1 m : 50 000 m
    b) 49 ha
    c) $18 300

16. **a), b)**, e.g., If kitchen is about 10 ft by 20 ft and scale diagrams are drawn on 8.5 in. by 11 in. paper, scale factor could be $\frac{1}{48}$.
    c) e.g., Estimate or measure the open floor space areas in each diagram and compare.

17. e.g., The area is divided by 4 in process A.

18. 81%

19. about 46¢ more

Lesson 8.5, page 497

1. **a)** similar
   b) similar
   c) similar
   d) not similar

2. **a)** Yes, all spheres are similar.
   b) i) $\frac{25}{22}$ **ii)** $\frac{22}{25}$

3. length: 52 m; beam: 8.5 m; height 43 m

4. **a)** Yes, all dimensions are proportional.
   b) S: 16 cm; L: 32 cm

5. length: 8.6 m; height: 3.8 m

6. 1.41 m by 1.68 m by 3.71 m

7. **a)** 2 **b)** length: 180 cm; height: 150 cm

8. **a)** $\frac{1}{3}$ **b)** 10 in.

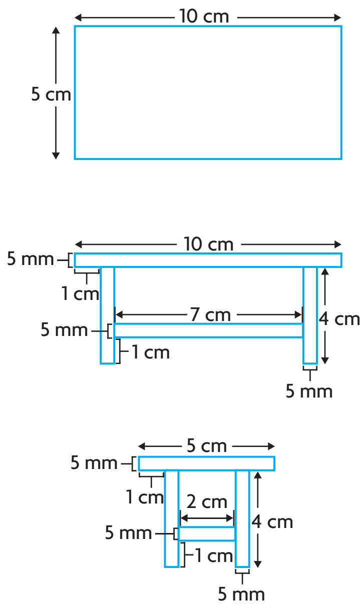
9. 16 in. by 2 in.

10. **a)** about 3 in. by 5 in. by 2 in.
    b) $\frac{87}{160}$
    c) $3\frac{1}{4}$ in. long, $4\frac{5}{8}$ in. high, $2\frac{1}{8}$ in. wide

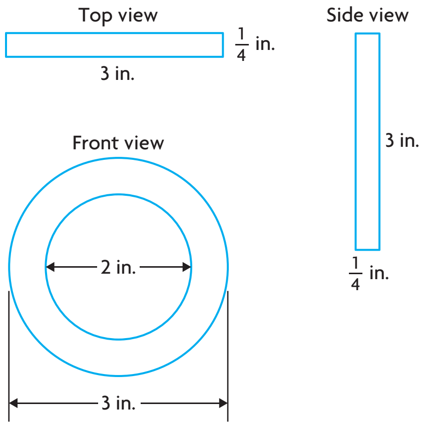
11. 2

12. e.g.,
    a) 6 m tall, 5 m wide
    b) Measure the metre stick in the photo to determine the scale factor. Then multiply the building measurements by the scale factor to determine the building dimensions in metres.

13.



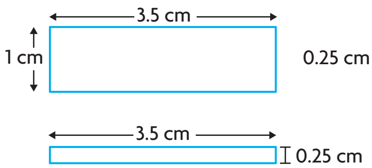
14.



15. e.g., Using a scale factor of $\frac{1}{20}$, the views would have the following dimensions:

Top view rectangle: 6.7 cm by 3.3 cm
Side view rectangle: 3.3 cm by 4.4 cm
Front view rectangle: 6.7 cm by 4.4 cm

16. e.g., for an eraser measuring 7.0 cm by 2.0 cm by 0.5 cm, using a scale factor of $\frac{1}{2}$:



17. **a)** no; The area of the base increases by the square of the scale factor.
b) no; The volume increases by the cube of the scale factor.

18. e.g., Both involve multiplying each dimension by a scale factor; shapes have two dimensions while objects have three dimensions.

19. **a)** 2.25
b) \$0.02

20. 25 cm^2

Lesson 8.6, page 508

1. **a)** **i)** 4 **ii)** 8
b) **i)** $\frac{9}{4}$ **ii)** $\frac{27}{8}$
c) **i)** 16 **ii)** 64
d) **i)** $\frac{25}{9}$ **ii)** $\frac{125}{27}$

2. **a)** 50 **b)** 2500 **c)** 125 000

3. 480 cm, 7650 cm^2

4. 864 m^3

5. **a)** 4500 cm^2
b) 9; The thickness of the paper will not change.

6. 3974 cm^3

7. **a)** 3.375 **b)** 2.25 **c)** 1.5

8. 9 and 27

9. 6600 cm^3

10. **a)** 1750 **b)** 25

11. **a)** 2.7 cm **c)** 13.5
b) 3.7 **d)** 49.5

12. 2.7

13. 72%

14. **a)** 1.5 **c)** 3.375
b) 2.25 **d)** 988 cm^3, 3334 cm^3

15. **a)** No, it will take about four times as much $(k = 2; k^2 = 4)$.
b) No, it is $\frac{1}{8}$ the volume of the large shoebox $\left(k = \frac{1}{2}; k^3 = \frac{1}{8}\right)$.

16. **a)** Surface area of scaled cylinder $= k^2(2\pi r^2 + 2\pi rh)$
Volume of scaled cylinder $= k^3(\pi r^2 h)$
b) Surface area of scaled cone $= k^2(\pi r^2 + \pi rs)$
Volume of scaled cone $= k^3\left(\frac{1}{3}\pi r^2 h\right)$

17. e.g., Consider the relationship between the volumes. The scale factor is 2, so the larger prism has a volume that is 8 times the volume of the smaller prism. Eight of the smaller prisms will fit inside the larger prism.

18. 9

19. **a)** 37 914 864 km^2 **b)** 21 952 700 000 km^3

20. e.g., \$20.16, assuming the heights are the same and that frosting costs the same as the interior of the cake.

Chapter Self-Test, page 512

1. **a)** increasing: 1993–1995, 1996–2000, 2001–2004, 2005–2008; decreasing: 1995–1996, 2000–2001, 2008–mid-2009
b) 2004–2005
c) 1998–1999; 2007–2008
d) e.g., general economic conditions

2. 1250 L

3. 1 m by 1.5 m

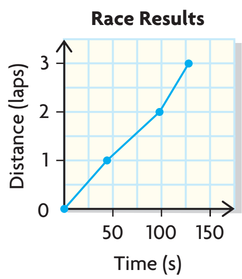
4. **a)** 2.4 cm, 6.8 cm **c)** 41.7 cm^2
b) 7.2 cm **d)** 54.7 cm^3

5. 3.5 ft

6. **a)** no
b) The largest pizza is the best buy.

Chapter Review, page 515

1.



2. **a)** The second rate (\$2.26/kg) is lower.
b) The second rate (29 km/h) is lower.
c) The first rate is lower (the second is 6.8 L/100 km).
d) The first rate (36 km/h) is lower.

3. Yes, her projected time is under 2 h 39 min.

4. **a)** about 30 sq yards
b) locally (she will save about \$10)

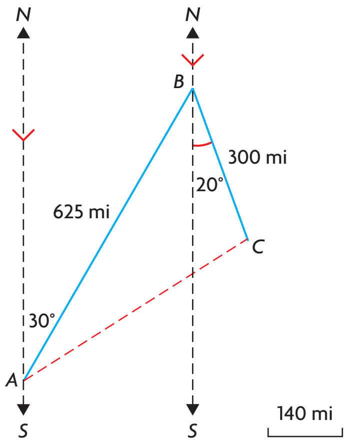
5. $\frac{3}{200}$

6. e.g.,

a) $\frac{1}{2}$

b) for a book cover, a rectangle 10.5 cm by 13.2 cm

7. **a)**



b) Use a ruler to measure the distance from C to A on the scale drawing in part (a); then use the scale to calculate the actual distance.

8. **a)** 96%

b) 18%

c) e.g., Many marketers stretch the truth to make themselves look as good as possible, and if the pizzeria owners are like this, then they probably mean (b).

9. **a)** about 309 m^2 **b)** 2.84

10. **a)** 5.5 in. by 7.7 in. **b)** 21%

11. 2.5 cm

12. **a)** 6 **b)** 9 m

13. 2 ft 2 in., 1 ft 7 in., 5.4 in.

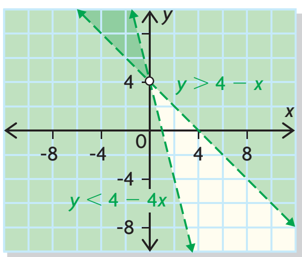
14. 14 580 cm^3

15. 403 cm^2

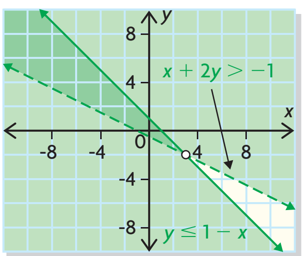
16. 40 mm by 76 mm by 8 mm

Cumulative Review, Chapters 6–8, page 520

1. **a)**



b)



2. No; the second inequality is not satisfied.

3. **a)** Let x represent the number of mountain bikes, and let y represent the number of racing bikes.

$\{(x, y) \mid 3x + 2y \leq 120, x \in W, y \in W\}$

$\left\{(x, y) \,\middle|\, \frac{x}{4} + \frac{y}{2} \leq 20, x \in W, y \in W\right\}$

Objective function: $P = 250x + 210y$

b) 20 mountain bikes and 30 racing bikes

4. **a)** **i)** $f(x) = x(x - 8)$

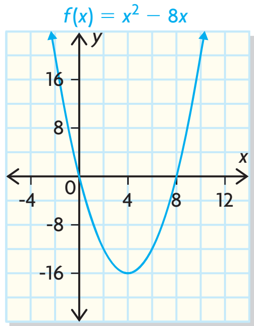
x-intercepts: (0, 0), (8, 0)

axis of symmetry: $x = 4$

vertex: (4, −16)

y-intercept: (0, 0)

ii)



iii) domain: $x \in R$

range: $y \geq -16, y \in R$

b) **i)** $y = (x + 5)(x - 3)$

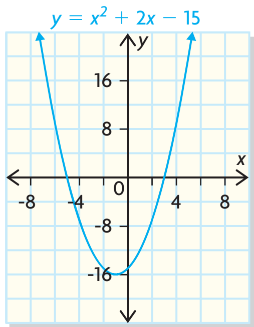
x-intercepts: (−5, 0), (3, 0)

axis of symmetry: $x = -1$

vertex: (−1, −16)

y-intercept: (0, −15)

ii)

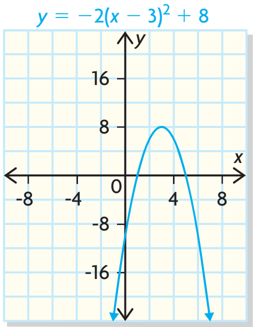


iii) domain: $x \in R$

range: $y \geq -16, y \in R$

5. **a)** **i)** axis of symmetry: $x = 3$
vertex: $(3, 8)$
y-intercept: $(0, -10)$

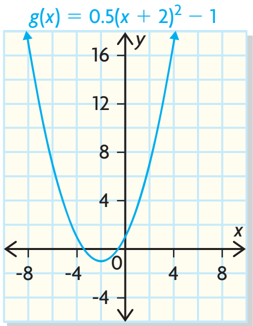
ii)



iii) domain: $x \in \mathrm{R}$
range: $y \leq 8, y \in \mathrm{R}$

b) **i)** axis of symmetry: $x = -2$
vertex: $(-2, -1)$
y-intercept: $(0, 1)$

ii)



iii) domain: $x \in \mathrm{R}$
range: $y \geq -1, y \in \mathrm{R}$

6. **a)** up, because the coefficient of x^2 is positive
b) $(-3, -5)$
c) a minimum value, because it opens up

7. $y = -\frac{1}{4}(x + 4)(x - 8)$; $y = -\frac{1}{4}x^2 + x + 8$

8. \$13

9. **a)** $-3, 3$ **b)** $-\frac{1}{2}, 3$

10. **a)** $-\frac{1}{2}, 5$
b) $-4, 8$
c) $-\frac{2}{3}, 4$
d) $3 \pm \sqrt{19}$
e) $1, 5$
f) $\frac{61 \pm \sqrt{3061}}{30}$

11. **a)** 4375 m **b)** 24.5 s

12. 60 min

13. **a)** 1500 m by 2500 m **b)** 375 ha

14. 65 m by 108 m by 215 m