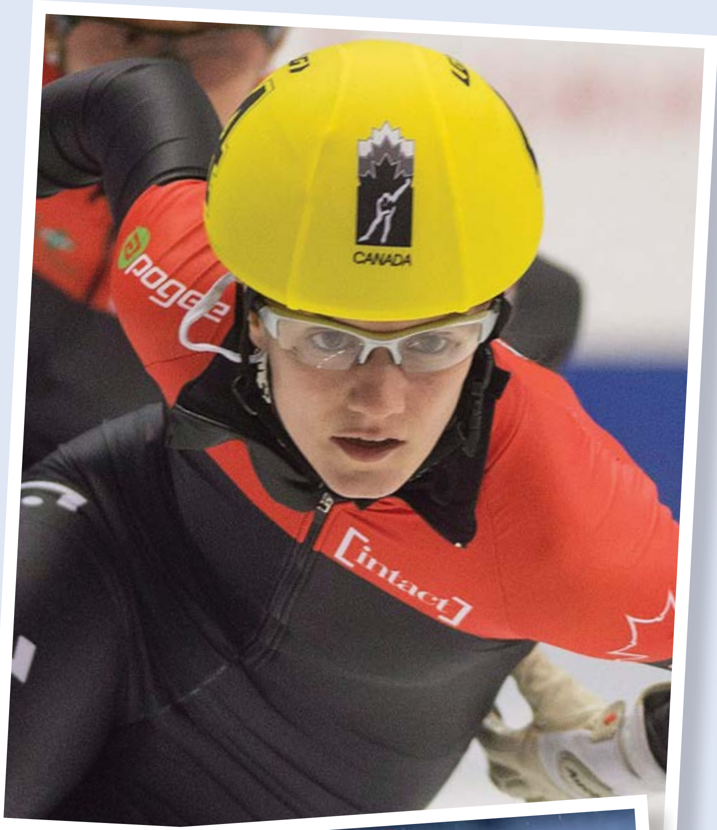
Chapter

7

Ratios, Rates, and Proportional Reasoning



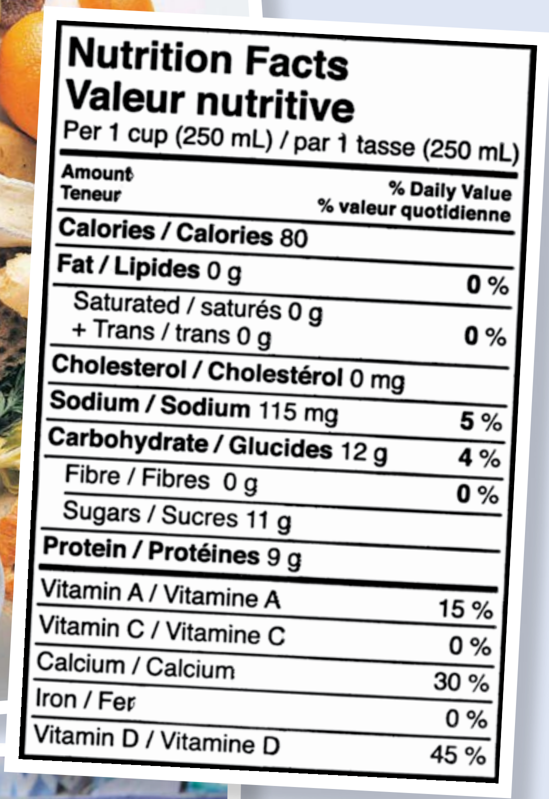
Jessica Hewitt is a short track speed skater from Langley, British Columbia. She has won medals at several World Championships as well as the 2014 Olympic Winter Games. It takes intense training and good nutrition to succeed as an elite athlete. Dietitians, coaches, and other sports professionals use proportional reasoning to help athletes achieve their goals. For example, dietitians plan meals and snacks to ensure athletes eat food that provides energy, carbohydrates, protein, vitamins, and other nutrients in specific ratios. This helps the athletes maintain their body weight and fitness level. In training sessions, coaches work with skaters like Jessica to improve their speed and stamina.

Big Idea

You can use ratios, rates, and proportional reasoning to compare quantities.

Inquire and Explore

- How can two quantities be compared, represented, and communicated?
- How are fractions and ratios interrelated?
- How does ratio use in mechanics differ from ratio use in architecture?



Nutrition Facts
Valeur nutritive
Per 1 cup (250 mL) / par 1 tasse (250 mL)

Amount / Teneur	% Daily Value / % valeur quotidienne
Calories / Calories 80	
Fat / Lipides 0 g	**0 %**
Saturated / saturés 0 g + Trans / trans 0 g	**0 %**
Cholesterol / Cholestérol 0 mg	
Sodium / Sodium 115 mg	**5 %**
Carbohydrate / Glucides 12 g	**4 %**
Fibre / Fibres 0 g	**0 %**
Sugars / Sucres 11 g	
Protein / Protéines 9 g	
Vitamin A / Vitamine A	15 %
Vitamin C / Vitamine C	0 %
Calcium / Calcium	30 %
Iron / Fer	0 %
Vitamin D / Vitamine D	45 %

(top left) Jessica Hewitt skates to a first place finish in the women's 500-metre final at the 2014 Canadian short track speed skating team selections.

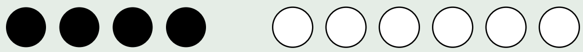
(bottom left) Marianne Saint-Gelais, Valerie Maltais, Jessica Hewitt, and Marie-Eve Drolet receive silver medals at the 2014 Winter Olympics.

(bottom right) Lucas Makowsky and his coach, Marcel Lacroix analyze video footage at a training session.

Get Ready

Writing Ratios

A **ratio** compares two quantities. The order of the words in a sentence shows the order of the numbers in the ratio. You can write ratios in several ways.



To express the ratio of black balls to the total number of balls, you can use

- *Words*: four compared to ten, or 4 to 10
- *Ratio Notation*: 4 : 10
- *Fraction*: $\frac{4}{10}$ or $\frac{2}{5}$

You can only write a ratio as a fraction if you are comparing part of the group to the whole group.

1. Use the diagram above to write each ratio. Express each answer in two different ways.

a) black balls to white balls

b) white balls to black balls

c) white balls to total number of balls

2. What does each of the following ratios represent in the diagram above?

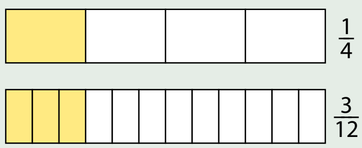
a) 6 : 4 **b)** 6 : 10 **c)** 5 : 2

3. a) Create a ratio of your choice using two quantities.

b) Show how to represent your ratio in several ways.

Equivalent Fractions

The fractions $\frac{1}{4}$ and $\frac{3}{12}$ are **equivalent fractions**. They are different ways of expressing the same value.



$$\underset{\div 3}{\overset{\div 3}{\frac{3}{12} = \frac{1}{4}}} \qquad \underset{\times 3}{\overset{\times 3}{\frac{1}{4} = \frac{3}{12}}}$$

Two fractions are equivalent if the numerators and denominators are related by the same multiplier or divisor.

4. Are the fractions equivalent? Show how you know.

a) $\frac{2}{3}$ and $\frac{4}{6}$ **b)** $\frac{2}{10}$ and $\frac{3}{15}$

c) $\frac{5}{6}$ and $\frac{25}{36}$ **d)** $\frac{10}{25}$ and $\frac{2}{5}$

5. List two equivalent fractions for each fraction.

a) $\frac{1}{4}$ **b)** $\frac{6}{16}$ **c)** $\frac{4}{12}$ **d)** $\frac{2}{11}$

6. Identify the missing value to make an equivalent fraction. Show how you calculated each value.

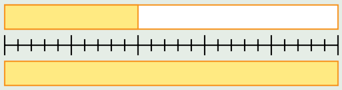
a) $\frac{5}{8} = \frac{■}{24}$ **b)** $\frac{2}{3} = \frac{4}{■}$

c) $\frac{1}{3} = \frac{5}{■}$ **d)** $\frac{16}{28} = \frac{■}{7}$

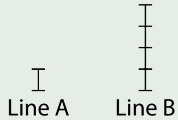
Comparing Quantities

A fraction can represent part of a whole.

The top line is $\frac{2}{5}$ or $\frac{10}{25}$ of the bottom line.

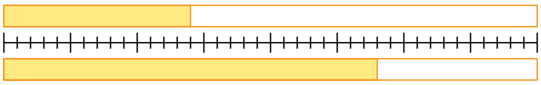


You can use a **multiplier** to compare two quantities. In the diagram, Line B is 4 times as long as Line A. The multiplier of Line B compared to Line A is 4, or the ratio 4 : 1.

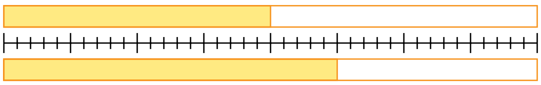


7. Give three equivalent fractions that compare the top line to the bottom line.

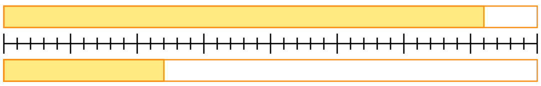
a)



b)

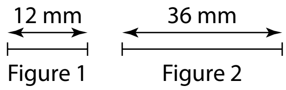


c)

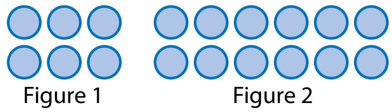


8. What is the multiplier from Figure 1 to Figure 2?

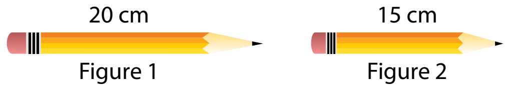
a)



b)



c)



Changing Between Percents and Fractions

To convert a **percent** to a fraction, you can write it as a fraction out of 100. The percent is the numerator and the denominator is 100. Sometimes you can reduce the fraction to lowest terms.

$43\% = \frac{43}{100}$ numerator / denominator

$$70\% = \frac{70}{100} \overset{\div 10}{\underset{\div 10}{=}} \frac{7}{10}$$

$$28\% = \frac{28}{100} \overset{\div 4}{\underset{\div 4}{=}} \frac{7}{25}$$

To change a **fraction** to a percent:

- use equivalent fractions

$$\frac{13}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{65}{100}$$
$$= 65\%$$

or

- divide the numerator by the denominator

$\frac{13}{20} = 13 \div 20$ Divide.
$= 0.65$
$= 0.65 \times 100\%$ Multiply by 100%.
$= 65\%$ Add a percent symbol.

9. Write each percent as a fraction in lowest terms.

a) 62% **b)** 40% **c)** 15% **d)** 24%

10. Change each fraction to a percent.

a) $\frac{17}{50}$ **b)** $\frac{8}{25}$ **c)** $\frac{9}{10}$ **d)** $\frac{7}{20}$

7.1

Ratios

Focus On . . .

In this lesson, I will learn to

- represent two-term and three-term ratios in a variety of ways
- represent part-to-part and part-to-whole ratios in a variety of ways
- identify and describe ratios used in real life
- analyze situations and solve problems using ratios

Ratios compare quantities. How can you use ratios to enlarge or reduce images?

Explore and Analyze

1. Look at the photo and its enlargement above. Consider and discuss the following:

- How might you compare the size of the images mathematically? What measurements would you need to make?
- How would it change things if the larger photo was the "original" and the smaller was a reduction?

2. Draw a shape of your choice by hand or using technology. Then, draw an enlargement or reduction of the shape. Consider and discuss the following:

- How does the size of your second image compare to your first?
- Is the size comparison between the two drawings the same for all parts of the drawing? Explain your reasoning.
- What is the best way to express this comparison mathematically? Why?

3. How can you use ratios to express size comparisons of enlarged and reduced images? Are there other ways to express size comparisons mathematically?

Develop Understanding

Example 1: Represent Ratios

Ann is making a button blanket, a type of ceremonial robe used by First Peoples of coastal British Columbia. For one part of the design she is planning to use a total of 20 buttons, including 3 different sizes of white buttons.

small medium large

a) Write a **two-term ratio** to compare the number of small buttons to the number of medium buttons.

b) Compare the number of large buttons to the total number of buttons. Use ratio notation and express the ratio as a fraction in lowest terms.

c) What comparison of buttons does the ratio 4 : 10 represent?

d) Use a **three-term ratio** to compare the large, medium, and small buttons.

Solution

a) You can use a **part-to-part ratio** to compare the 10 small buttons and 6 medium buttons.

Method 1: Represent Using a Diagram

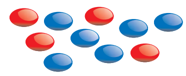
:

You can also show this ratio as an equivalent ratio in reduced form. Group each part of the ratio into equal-sized smaller groups.

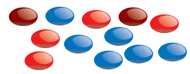
10 small buttons for 6 medium buttons is equivalent to 5 small buttons for 3 medium buttons.

for each

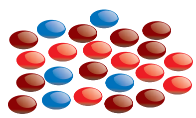
two-term ratio
- a comparison of two numbers
- written as $a:b$ or a to b
- blue : red is 6 : 4



three-term ratio
- a comparison of three numbers
- written as $a:b:c$ or a to b to c
- blue : red : brown is 6 : 4 : 2



part-to-part ratio
- compares different parts of a group to each other
- 10 : 8 is the part-to-part ratio of brown to red beads



Method 2: Represent Using Symbols or Words

The ratio of small buttons to medium buttons is 10 to 6 or 10 : 6.

You can divide to write an equivalent ratio in reduced form, since the terms have a common factor.

$$\begin{array}{c} 10:6 \\ \div 2 \downarrow \quad \downarrow \div 2 \\ 5:3 \end{array}$$

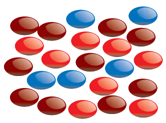
Why is this a part-to-part ratio?

10 small buttons for 6 medium buttons is equivalent to 5 small buttons for 3 medium buttons.

You cannot write this ratio as a fraction because it is a part-to-part ratio. You are comparing part of the group to another part of the group, not to the whole group.

part-to-whole ratio

- compares one part of a group to the whole group
- 5 : 23 is the part-to-whole ratio of blue to total number of beads
- can be expressed as a fraction, decimal, or percent



b) You can use a **part-to-whole ratio** to compare the 4 large buttons out of the total of 20 buttons.

Express the ratio as 4 to 20 or 4 : 20.

Four out of every 20 buttons are large.

You can write a part-to-whole ratio as a fraction because you are comparing part of the group to the whole group.

$$\frac{\text{large}}{\text{total}} = \frac{4}{20}$$

Why is this a part-to-whole ratio?

Write an equivalent fraction in lowest terms.

$$\frac{4}{20} \overset{\div 4}{\underset{\div 4}{=}} \frac{1}{5}$$

What information is lost when you write this fraction in lowest terms? Is it important information?

Four large buttons for 20 total buttons is equivalent to 1 large for every 5 of the total.

c) There are 4 large and 10 small buttons, so 4 : 10 represents large to small.

d) You can compare large, medium, and small buttons using a three-term ratio.

large : medium : small = 4 : 6 : 10

To write in lowest terms, divide all three terms by the same number.

$$\begin{array}{c} 4:6:10 \\ \div 2 \downarrow \quad \downarrow \div 2 \quad \downarrow \div 2 \\ 2:3:5 \end{array}$$

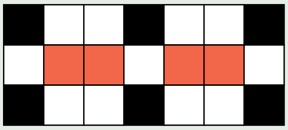
How can you tell when a three-term ratio is in lowest terms?

Show You Know

a) What is the ratio of red tiles to total titles?

b) What tiles have a ratio of 2 : 3?

c) What is the ratio of red to black to white tiles?



Example 2: Use Ratios to Determine Values and Analyze Situations

A natural food store makes its own trail mix. For one variety, they use nuts, dried fruit, and chocolate candy in a ratio of 12 : 9 : 4 by mass.

a) What is the ratio of fruit to nuts, in lowest terms?

b) The store's owner says that the mix is more than 15% chocolate candy by mass. Is she correct?

c) If the store makes 150 kg of the mix in a month, how much of each ingredient do they need?

Solution

a) The ratio of nuts to fruit to chocolate candy is 12 : 9 : 4, so fruit : nuts = 9 : 12.

Write the ratio in lowest terms using a common factor.

$$\begin{array}{ccc} & 9:12 & \\ \div 3 \swarrow & & \swarrow \div 3 \\ & 3:4 & \end{array}$$

Is this a part-to-part or a part-to-whole ratio?

The ratio of fruit : nuts is 3 : 4. The mix consists of 3 parts fruit for every 4 parts nuts.

b) Determine the ratio of chocolate candy to total ingredients. Add the terms representing each ingredient to find the total.

$$\begin{aligned}\text{Total} &= 12 + 9 + 4 \\ &= 25\end{aligned}$$

$$\frac{\text{chocolate candy}}{\text{total}} = \frac{4}{25}$$

Is this a part-to-part or a part-to-whole ratio?

Four of every 25 parts of the mix are chocolate candy. Change the fraction to a percent.

Method 1: Use a Fraction

$$\frac{4}{25} \underset{\times 4}{\overset{\times 4}{=}} \frac{16}{100}$$

$$= 16\%$$

Method 2: Change to a Decimal

$$\begin{aligned}\frac{4}{25} &= 0.16 \\ &= 0.16 \times 100\% \\ &= 16\%\end{aligned}$$

4 $\boxed{\div}$ **25** $\boxed{=}$ **0.16**

The mix is 16% chocolate candy. The store's owner is correct that the mix is more than 15% chocolate candy.

c) The 25 kg of trail mix is made up of 12 kg of nuts, 9 kg of fruit, and 4 kg of chocolate candy. To make 150 kg of mix, you need 6 times as much, since $150 \div 25 = 6$.

You can use a ratio table to help organize the information:

Nuts	Fruit	Chocolate Candy	Total
12	9	4	25
×6 72	×6 54	×6 24	×6 150

The store needs to use 72 kg of nuts, 54 kg of fruit, and 24 kg of chocolate candy to make 150 kg of the mix.

Show You Know

A company makes fruit smoothies using a ratio of 4 : 3 : 8 of fruit purée to yogurt to milk.

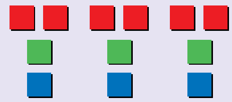
a) What is the ratio of milk to fruit purée, in lowest terms?

b) The company claims that their smoothies are more than 25% fruit. Are they correct?

c) If the store makes a total of 4500 L of smoothies in a week, how much of each ingredient do they need?

Connect and Reflect

Key Ideas

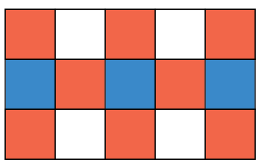


- A two-term ratio compares two quantities.
  red squares to blue squares is 6 : 3
- A three-term ratio compares three quantities.
  red : blue : green is 6 : 3 : 3
- A part-to-part ratio compares different parts of a group.
  red : green is 6 : 3
- A part-to-whole ratio compares one part of a group to the entire group.
  blue : total is 3 : 12
- You can represent ratios using words, symbols, or pictures.
- You can write a part-to-whole ratio as a fraction, decimal, or percent.
- You can write an equivalent ratio by dividing or multiplying each term by the same number.

Practise

For help with #1 to #4, refer to Example 1 on pages 223–224.

1. Use the tile pattern to answer the questions.



a) What is the ratio of blue tiles to white tiles?

b) What is the ratio of red to white to blue tiles?

c) What tiles can be represented with a ratio of 1 to 2? Show how you know.

d) What is the ratio of blue tiles to total tiles? Write your answer in lowest terms using ratio notation and as a fraction.

2. Tyler counts 20 cars in the school parking lot. Of these, 6 are red, 4 are green, and 1 is yellow.

a) Draw a diagram to represent the situation.

b) How many cars are not red, green, or yellow?

c) What is the ratio of yellow to green to red cars?

d) What is the ratio of red to total cars? Write the ratio as a fraction and as a percent.

3. Write each ratio using ratio notation. Then, rewrite each ratio in lowest terms.

a) $2 compared to $8

b) the width of the cover of this book compared to its length, in centimetres

c) the ratio of boys to girls to total students in a class where 14 of 30 students are girls

4. Write each ratio as a fraction and a percent.

a) You spend $4 out of $10.

b) A team wins 3 games and loses 6 games. What is the ratio of games won to games played?

c) A bag contains 12 red and 3 blue beads. Compare blue beads to total beads.

For help with #5 to #7, refer to Example 2 on pages 225–226.

5. Tamara is making fruit punch using the recipe shown.

Tamara's Fruit Punch
2 parts orange juice concentrate
1 part raspberry juice concentrate
1 part lemon juice
12 parts soda water

a) What is the ratio of juice concentrate to soda water, in lowest terms?

b) Tamara thinks the punch will be more than 5% raspberry juice concentrate. Is she correct? How do you know?

c) If Tamara makes 1600 mL of punch, how much of each ingredient will she need?

6. Ava bought a 250-mL bottle of concentrated cleaner. It must be mixed with water in a 1 : 7 ratio before it can be used. She plans to mix it in a 400-mL spray bottle.

a) How much cleaner does Ava need to add to the bottle before she fills it with water?

b) She thinks the mixed cleaner is less than 75% water. Is she correct? Show how you know.

c) If Ava uses all the cleaner, how many full spray bottles can she make?

7. Use the data to answer the questions.

Sport	Wins	Losses
Soccer	9	6
Volleyball	6	10
Basketball	12	8

a) Which sports have equivalent win–loss ratios? Show how you know.

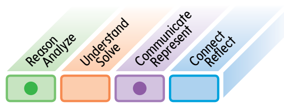
b) What is the ratio of wins to total games played for soccer? Give your answer as a fraction, a decimal, and a percent.

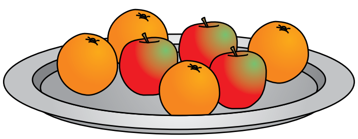
c) If the volleyball team plays 40 games in total for the season, how many wins might they expect based on their play so far?

d) Early in the season, the athletic coordinator predicted that the school's overall winning percent would be more than 60%. Was this prediction accurate? Show how you know.

Apply

8. Janine uses a ratio to compare the number of oranges to the number of apples.

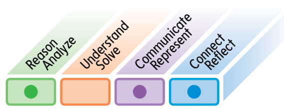




a) How does she know whether to write 3 : 4 or 4 : 3?

b) Would it make sense to write this ratio as a fraction? Why or why not?

9. ✓ **Competency Check** Use a diagram and your own example to explain how the fraction $\frac{2}{5}$ can be interpreted as a part-to-whole ratio.



10. A beaker contains 2 liquids that do not mix. Instead, they form layers as shown.

a) What part-to-part ratio(s) could you write using this situation?

b) What part-to-whole ratio(s) could you write using this situation?

c) Which ratios from parts a) and b) are appropriate to write as fractions and percents? Explain why. Then, determine those percents.

11. A lacrosse team played 28 games and won 4 out of every 7 games. There were no games that ended in a tie.

a) How many games did they lose?

b) What is the team's win–loss ratio?

c) If this trend continues, how many losses would you expect the team to have once they have won 20 games?

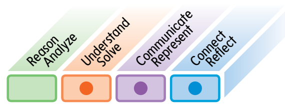
12. Sarah is attending a family reunion. Three eighths of the 48 people at the reunion are younger than 16 years old.

a) Does the statement represent a part-to-part or part-to-whole ratio? Explain.

b) Rewrite the information given in this situation using the word *ratio*.

c) How many people are 16 or older? Explain your thinking.

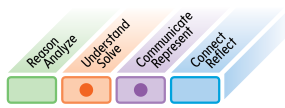
13. Diana and John are making three-cheese lasagna. The recipe calls for 100 g of Romano, 300 g of mozzarella, and 250 g of ricotta.



a) Write a ratio in lowest terms to compare the amounts of the 3 cheeses. State the order of the cheeses.

b) What amounts of Romano and ricotta do you need to make a lasagna that contains 900 g of mozzarella cheese?

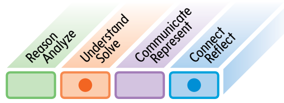
14. There are 48 passengers on a transit bus. At the next stop, 16 passengers get off and 12 new passengers get on the bus.



a) What is the ratio of the passengers who get off the bus compared to the original number on the bus?

b) What is the ratio of new passengers on the bus compared to the new total that are now on the bus?

c) Which ratios from parts a) and b) would be appropriate to write as percents? Explain why. Then, determine these percents to the nearest whole number.



15. Concrete mix is made by mixing cement powder, sand, and gravel in a ratio of 1 : 2 : 4 before the mix is combined with water.

a) Joseph has 50 kg of sand. How much of the other two ingredients does he need to add?

b) Joseph has 12 bags of cement powder, where each one contains 25 kg. How much mix in total can he make? How much of the other two ingredients does he need to add?

16. Not all flags have the same ratio of dimensions. The table shows the official height-to-length ratio for the flags of three North American countries.

Country	Ratio (height : length)
Canada	1 : 2
Mexico	4 : 7
United States	10 : 19

a) A Canadian flag on Gyp Mountain near Falkland, BC, is the largest in Western Canada. If it is 17 m long, what is its height?

b) The Peace Arch monument at the Douglas border crossing south of Vancouver, BC, has a Canadian and a U.S. flag on top. If both flags are 120 cm high, which one is longer?

c) Which of the three countries has a flag that is the closest to being a square? Explain your thinking.

17. A 30-kg bag of fertilizer is labelled 15–20–10. This means that it contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. How many kilograms of nitrogen, phosphorus, and potassium are in the bag?

18. Cam has a yard trimmer that runs on a mixture of oil and gas. The instructions say that the mixture of oil to gas needs to be 1 : 40.

a) Cam has an 80-mL bottle of oil. How much gas does he need to mix with it?

b) How could this instruction be given using a percent instead of a ratio?

c) If he fills the 200-mL tank with the mixture, how much of it will be oil?

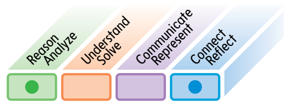
Extend

19. The golden rectangle is used often in art and architecture. For example, the front of the Parthenon, a temple in Athens, Greece, fits into a golden rectangle. A golden rectangle has a length-to-width ratio called the golden ratio, which is approximately 1.62 : 1.

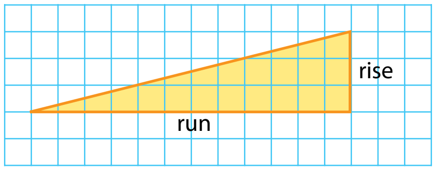
a) Which of these dimensions of rectangles are examples of golden rectangles?

- 24 m × 38.9 m
- 52 cm × 120.5 cm
- 348 mm × 565 mm

b) If the width of a golden rectangle is 6.4 m, what is its length? Give your answer to the nearest tenth of a metre.



20. The side view of a ramp is shown.



a) Express the ratio of rise to run in lowest terms. This ratio describes the slope of the ramp.

b) Express the slope ratio as a fraction, a decimal, and a percent.

c) Predict what effect each of the following would have on the slope of the ramp:

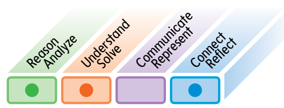
i) increasing the rise

ii) decreasing the rise

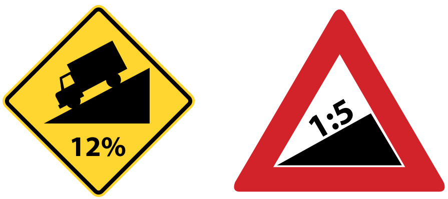
iii) increasing the run

iv) decreasing the run

d) Verify your predictions in part c).



21. ✓ **Competency Check** In North America and in many other places in the world, the slope of a road is given as a percent, as shown in the yellow sign. In some other places in the world, the slope of a road is indicated by a ratio, as shown in the red and black sign.



a) Which of these two roads is steeper?

b) Which type of sign do you think is more effective to show drivers how steep a hill is? Justify your decision.

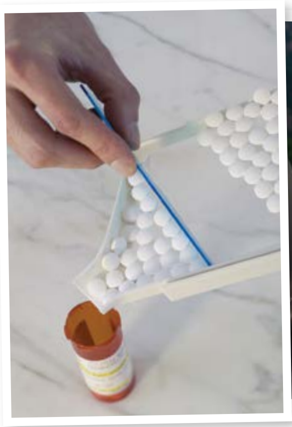
7.2

Rates

Focus On ...

In this lesson, I will learn to

- represent rates in a variety of ways
- identify, describe, and record rates used in real life
- use rates to analyze situations and solve problems



Rates are often used to relate 2 different quantities. For example,

- A car has a speed of 60 km/h or a fuel consumption rate of 8.5 L/100 km.
- A doctor measures your heart rate as 72 beats/min, or prescribes a medication at a rate of 50 mg/day.
- A plant grows 5 cm/month, the feeding rate for a calf is 4 L/day, or the price of strawberries is \$7/kg.

How is the information used in these rates useful?

rate

- the ratio of 2 measurements with different units
- can be expressed as a fraction that includes the 2 different units
- for example, a rate for purchasing bulk food is \$1.69 per 100 g or \$1.69/100 g or $\frac{\$1.69}{100\text{ g}}$

Explore and Analyze

1. Work with a small group.
   - Each person needs 6 small slips of paper. On each slip of paper, write a whole-number quantity between 2 and 20 along with an object and/or unit, such as "6 litres of milk," "3 cats," or "20 pieces of paper."
   - Fold all the slips of paper in half, and mix them up.
   - Choose a pair of slips of paper without looking. Open them and read them to yourself.
   - Write a rate using the quantities and objects/units on the slips you chose.
   - Write an equivalent **unit rate** for the rate that you wrote, including the appropriate units.
   - Choose two more pairs of slips. For each pair, write a rate and an equivalent unit rate.
   - Share your results with your group. For each of your rates, explain why you chose the order that you did to compare the two quantities.

unit rate

- a rate in which the second term is a single unit of measure, often 1
- for example, 64 beats per minute or 64 beats/min
- for example, \$3.46 per litre or \$3.46/L

2. Reflect and then write about your findings and discussion.
   - Which rate is the most realistic in representing something that could occur in real life? Which is the least realistic? Explain your choices.
   - Is there more than one way to write each rate? Explain.
   - What strategy or strategies did you use to convert rates to unit rates?
   - How did you know what units each unit rate should have?

Develop Understanding

Example 1: Determine a Unit Rate

Tyler rides his mountain bike on a 40-km trail. It takes him 2 hours and 30 minutes to complete the whole trail. What unit rate represents Tyler's average speed?

Solution

Speed is a unit rate that compares the distance travelled to the time it takes.

Write the time it takes Tyler to ride his bike in hours.

$\frac{30\text{ min}}{60\text{ min}} = 0.5\text{ h}$

So, 2 hours and 30 minutes is 2.5 hours.

Write a rate using the given values, and then divide to get a unit rate representing Tyler's average speed over the whole ride.

$$\begin{aligned}\text{Speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{40\text{ km}}{2.5\text{ h}}\\ &= 16\text{ km/h}\end{aligned}$$

Why is the final answer considered a unit rate?
How do you know what units to use in the answer?

Tyler's average speed was 16 km per hour.

Show You Know

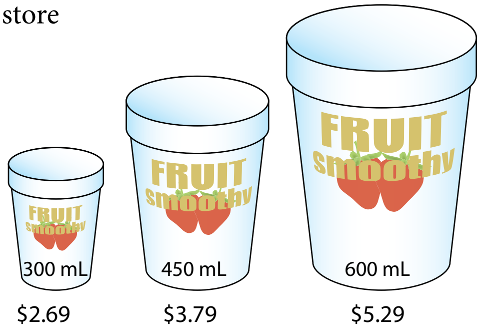
Miranda's family travels 390 km by car from Kelowna to Vancouver in 3 hours and 45 minutes without stopping. What average speed did they travel?

Example 2: Compare Prices Using Unit Rates

Brittany is buying a fruit smoothie. The store has the following cup sizes and prices.

a) Which size smoothie is the best buy according to price?

b) What else might Brittany consider when choosing a size?



Solution

a) ***Method 1: Compare Unit Rates in Millilitres Per Dollar***

Calculate the unit rate of smoothie volume per dollar for each container. Then, compare to see which is the *greatest*.

Small

300 mL for $2.69

$$\text{Unit rate} = \frac{\text{volume}}{\text{cost}}$$

$$= \frac{300 \text{ mL}}{\$2.69}$$

$\boxed{\text{C}}$ **300** $\boxed{\div}$ **2.69** $\boxed{=}$ **111.524164**

$$\approx 112 \text{ mL per dollar}$$

Medium

450 mL for $3.79

$$\text{Unit rate} = \frac{\text{volume}}{\text{cost}}$$

$$= \frac{450 \text{ mL}}{\$3.79}$$

$\boxed{\text{C}}$ **450** $\boxed{\div}$ **3.79** $\boxed{=}$ **118.733509**

$$\approx 119 \text{ mL per dollar}$$

Large

600 mL for $5.29

$$\text{Unit rate} = \frac{\text{volume}}{\text{cost}}$$

$$= \frac{600 \text{ mL}}{\$5.29}$$

$\boxed{\text{C}}$ **600** $\boxed{\div}$ **5.29** $\boxed{=}$ **113.421550**

$$\approx 113 \text{ mL per dollar}$$

The unit rate shows the volume of smoothie Brittany will get per dollar. Since the unit rate for the medium size is *greater* than the unit rates for the other two sizes, the best buy is the medium-sized smoothie for $3.79.

> Why is the *greatest* unit rate best when determining amount per dollar?

Method 2: Compare Unit Prices in Dollars Per 100 mL

Calculate the **unit price** per 100 mL for each size and then compare to see which is the *least*.

Small

\$2.69 for 300 mL, or 269¢ for 3 hundred mL

$$\text{Unit rate} = \frac{\text{cost}}{\text{volume}}$$

$$= \frac{269¢}{3 \text{ hundred mL}}$$

C 269 ÷ 3 = 89.6666666

$\approx$ 89.7¢ per hundred mL or 89.7¢/100 mL

Why is it more convenient to use cents than dollars here? Why is it more convenient to determine a rate per 100 mL than per mL?

Medium

\$3.79 for 450 mL, or 379¢ for 4.5 hundred mL

$$\text{Unit rate} = \frac{\text{cost}}{\text{volume}}$$

$$= \frac{379¢}{4.5 \text{ hundred mL}}$$

C 379 ÷ 4.5 = 84.2222222

$\approx$ 84.2¢ per hundred mL or 84.2¢/100 mL

Large

\$5.29 for 600 mL, or 529¢ for 6 hundred mL

$$\text{Unit rate} = \frac{\text{cost}}{\text{volume}}$$

$$= \frac{529¢}{6 \text{ hundred mL}}$$

C 529 ÷ 6 = 88.1666666

$\approx$ 88.2¢ per hundred mL or 88.2¢/100 mL

Why do you divide by 3, 4.5, and 6 instead of 300, 450, and 600?

The unit price for the medium size is less than the unit prices for the other two sizes. The best buy is the medium size for \$3.79.

Why is the *lowest* value the best when determining unit prices?

b) Brittany might consider how much money she has and how much smoothie she actually wants. A different size might be a better choice even though it has a higher unit cost.

unit price
- a special unit rate used when shopping
- often shown per litre, per kilogram, or per item
- sometimes shown per 100 g or per 100 mL
- makes it easier for shoppers to compare the cost of similar items

unit price

$$\frac{\$8.65}{2.5 \text{ L}} = \$3.46/\text{L}$$

Show You Know

Brett is buying potatoes. The store sells 2-kg bags for \$3.20 and 7-kg bags for \$10.85. The same type of potato is also sold in bulk for \$1.50/kg.

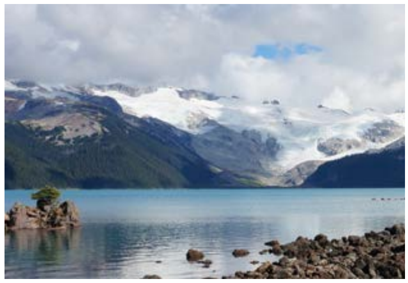
a) What is the best buy for potatoes at this store?

b) What else might Brett consider when making his decision?

Example 3: Use Unit Rates to Analyze a Situation

Scientists who study glaciers are called glaciologists. One measurement made by glaciologists is the rate at which the land area covered by glacial ice changes. The table shows estimates for the area covered by glaciers in Garibaldi Provincial Park, BC, at three different times.

Year	Area Covered by Glaciers (km^2)
1700	505
1988	297
2005	245



The rate at which glaciers are melting is increasing in Garibaldi Provincial Park. Do the data in the table support this statement?

Solution

Determine unit rates to compare the loss of ice for the two time periods given in the table. Compare the area lost to the number of years in the time period.

1700 to 1988

$$\text{Area lost} = 505 \text{ km}^2 - 297 \text{ km}^2 = 208 \text{ km}^2$$

$$\text{Time taken} = 1988 - 1700 = 288 \text{ years}$$

$$\text{Rate of loss of ice} = \frac{208 \text{ km}^2}{288 \text{ years}} \approx 0.7 \text{ km}^2/\text{yr}$$

1988 to 2005

$$\text{Area lost} = 297 \text{ km}^2 - 245 \text{ km}^2 = 52 \text{ km}^2$$

$$\text{Time taken} = 2005 - 1988 = 17 \text{ years}$$

$$\text{Rate of loss of ice} = \frac{52 \text{ km}^2}{17 \text{ years}} \approx 3.1 \text{ km}^2/\text{yr}$$

The average rate of loss of glacier ice for the more recent time period is 3.1 km^2/year. This value is far greater than the 0.7 km^2/year average rate for the earlier time period. Since the more recent rate is much higher, it supports the statement that the rate at which glaciers in the park are melting is increasing.

Show You Know

Bryanna works in a greenhouse that grows tomatoes. She measures the height of the plants to track their growth. The table shows some of her measurements.

Time Since Planting (days)	Height of Plant (cm)
8	2
30	17
90	84

She suspects that the plants grow more slowly earlier in their development. Do her measurements support this idea?

Connect and Reflect

Key Ideas

- A rate is the ratio of two measurements with different units.
- You can express a rate as a fraction that includes the two different units.
- A unit rate is a rate in which the second term is a single unit of measure, often 1.
- The units in the unit rate come from the two quantities being divided. For example, amount of rainfall measured in centimetres divided by months gives centimetres per month, or cm/mo.
- A unit price is a unit rate that makes it easier to compare the cost of similar items.

Practise

For help with #1 to #4, refer to Example 1 on page 233.

1. Determine the speed of each animal. Include units in your answer.

a) An orca swims 110 km in 2 h.

b) A blue whale travels 193 km in 10 h.

c) A Canada goose flies 800 m in 50 seconds.

d) A snail moves 40 mm in 5 min.

2. What is the unit rate in each case?

a) A black bear eats 22 kg of berries in 2 weeks.

b) Cathy plants 45 daffodils in 30 min.

c) A bank teller helps 65 customers in 2.5 h.

d) A water tank leaks 58 L of water in 4 days.

3. Ruby-throated hummingbirds and monarch butterflies travel similar paths across the Gulf of Mexico. The distance is just over 800 km. It takes the hummingbird 18.5 h and the monarch butterfly 41.6 h to cross the Gulf.

a) Estimate the speed of the hummingbird and the butterfly.

b) Calculate the speed of the hummingbird and the butterfly to the nearest tenth.

4. Gina earns $78.00 for working 6 h. Asad makes $192.50 for working 14 h. Who has the greater hourly rate of pay?

For help with #5 and #6, refer to Example 2 on pages 234–235.

5. Fraser is shopping for milk. It is available in three sizes.

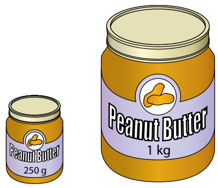
a) What is the unit price per 100 mL for each carton of milk?

b) Determine the unit price using different units than you used in part a).

c) Which carton of milk is the best buy? Explain how you know.

d) What other factors might Fraser consider when deciding which size to buy?

6. Mala is shopping for peanut butter. Her favourite brand is available in two sizes.



250 g $5.49 | 1 kg $7.99

a) Estimate which jar is the better buy. Explain your thinking.

b) Determine the better buy. Justify your answer mathematically.

c) Which approach (estimating or calculating) would you be more likely to use if you were actually shopping? Explain your choice.

For help with #7, refer to Example 3 on page 236.

7. The table shows some data that Brent has collected about the growth rates of polar bear cubs.

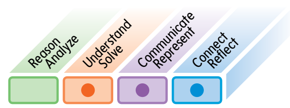
Birth Mass (kg)	Mass After 8 Weeks (kg)	Mass After 20 Weeks (kg)
0.6	6.2	12.2

Do the data support the statement that bear cubs grow faster immediately after birth compared to later on?

Apply

8. How is a rate different from a ratio? Explain using your own examples.

9. **Competency Check**

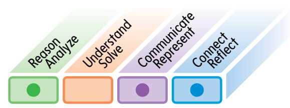


a) Give two examples of rates that are used in everyday life. Share your examples with a classmate.

b) What units measure each of the rates in part a)?

c) Explain why a rate cannot be expressed as a percent.

10. **Competency Check** Two brands of canned dog food are on sale. The cans are the same size. Brand A costs $13.60 for 8 cans and Brand B costs $8.75 for 5 cans.



a) Explain how you can compare the cost of the two brands of dog food. Determine the better buy.

b) Is the lowest cost always the best decision when making a purchase? What other factors could be considered in this case?

11. The table shows driving information for three drivers. There are two ways to use rates to express how the distance travelled compares to the fuel used in metric units: fuel economy, expressed in km/L, or fuel consumption, expressed in L/100 km.

Driver	Distance (km)	Fuel Used (L)
Joe	400	28
Sarah	840	60
Martin	245	20

a) What is the fuel economy for each vehicle in km/L? Give your answers to two decimal places.

b) What is the fuel consumption of each vehicle in L/100 km? Give your answers to two decimal places.

c) Which driver's vehicle has the best rate? Is it the higher value for both parts a) and b)? Explain.

12. Cara works for an airline and uses an app on her phone to check daily currency conversion rates. The screen shows the rates for some popular currencies on a particular day. She has $400 CDN to convert to another currency.

a) Use mental math to estimate the value of $400 in British pounds and in U.S. dollars.

b) How many euros can Cara get?

c) What is the money worth in Russian rubles?

d) Compare the rates given here to current conversion rates. Which places are better for a Canadian travelling abroad today?

13. Grace is an egg farmer who sells eggs in two different-sized containers: a dozen eggs for $5 or a 30-egg flat for $10.80. She is going to start selling an 18-egg carton as well and is wondering what price to charge. She wants to set the unit price of the 18-egg carton between the unit prices for the other two sizes. What price range can she choose from for the new size?

Extend

14. ✓ **Competency Check** Hardeep goes to the bank to get some U.S. dollars for a trip to Seattle. He pays $500 CDN and receives $379.15 USD. At the end of his trip, Hardeep returns to the bank and converts an unspent $180 USD back to Canadian dollars. He receives $245.50 CDN. Will Hardeep be happy with how the exchange rate has changed during his trip?

15. Daniel's yard is 10 m by 6 m. His push lawnmower can cut the grass in 15 minutes. The field behind Grace's house measures 30 m by 50 m. Her ride-on mower can mow the field in an hour. There is a 400-m^2 area between the two properties. How long will it take Daniel to mow this area? How long will it take Grace?

16. A planet takes one day to make one revolution of its axis. Think of each planet as a sphere. If you are standing on the equator of a planet, you are travelling in a circle as the planet spins on its axis. Use the table to find the rotation rate in kilometres per hour for a point on the equator of each planet.

Planet	Radius at Equator (km)	Length of Day (h)
Venus	6 051	2 808
Earth	6 378	24
Saturn	60 268	10 233

The formula relating the circumference, C, of a circle to its radius is $C = 2 \times \pi \times r$.

17. A farmer applies liquid fertilizer to a field at a rate of 2 L/m^2. What is this equivalent to in millilitres per square centimetre?

7.3 Using Proportional Reasoning

Focus On . . .

In this lesson, I will learn to

- represent proportional relationships in a variety of ways
- use proportional reasoning to determine unknown values
- analyze situations to determine whether a proportional relationship exists

You use proportional reasoning all the time without realizing it when you

- predict that it will take 15 minutes to walk 6 blocks if it took 5 minutes to walk 2 blocks
- assume that you will get 50 kg of peaches from 5 trees if 1 tree produced 10 kg
- double each ingredient in a recipe if you want twice the final product
- guess that it will take you 20 minutes to type 500 words if you took 2 minutes to type 50 words

How can you use a rate to make a prediction?

proportion

- a statement that says that two ratios or two rates are equivalent
- can be written in words: "3 km in 2 h is equivalent to 9 km in 6 h"
- can be written as an equation with two fractions:

$$\overset{\times 3}{\frac{3\text{ km}}{2\text{ h}} = \frac{9\text{ km}}{6\text{ h}}}_{\times 3}$$

proportional relationship

- a relationship in which one quantity is always related to another by a constant multiplier
- for example, if one quantity is tripled, then the other is also tripled

Explore and Analyze

1. Work with a partner to count the number of times each of you can snap your fingers in exactly 10 seconds. Count the snaps even if they do not make a noise.

Use your results to predict how many times you could snap your fingers in

a) one minute

b) one hour

c) one day

Justify your predictions mathematically.

2. Reflect and then write about your findings and discussion.

- Did you use a **proportion** to find your answers? If so, explain how. If not, explain how you could.
- Do you think there is always a **proportional relationship** between time and the number of times you can snap? Explain.

Develop Understanding

Example 1: Use Proportional Reasoning With Rates

Sam paints 15 m^2 of his fence in 3 hours. The total area of the fence is 90 m^2. If he continues to paint at the same rate, how long will it take him to paint the entire fence?

Solution

Method 1: Use Proportional Reasoning With Words

Write a statement that expresses the proportional relationship. Use reasoning to determine the multiplier and the unknown value.

"3 h to paint 15 m^2, so how many hours to paint 90 m^2?

90 m^2 is 6 times as much as 15 m^2, so the time should also be 6 times as much as 3 h.

$3\text{ h} \times 6 = 18\text{ h}$"

It will take Sam 18 h to paint the entire fence.

How can you tell that the multiplier is 6?
How does the multiplier help you determine the answer?

Method 2: Use Equivalent Ratios

Write a proportion to determine the unknown value.

$$\frac{3\text{ h}}{15\text{ m}^2} = \frac{\blacksquare}{90\text{ m}^2}$$

Solve the proportion by determining the multiplier.

$$\frac{3\text{ h}}{15\text{ m}^2} \underset{\times 6}{\overset{\times 6}{=}} \frac{\blacksquare}{90\text{ m}^2}$$

How does the proportion help you see that the multiplier is 6?

$3\text{ h} \times 6 = 18\text{ h}$

It will take Sam 18 h to paint the entire fence.

Method 3: Use a Unit Rate

Use a Unit Rate in m²/h

15 m² painted in 3 h can be expressed as $\frac{15\ \text{m}^2}{3\ \text{h}}$.

Determine the unit rate: $\frac{15\ \text{m}^2}{3\ \text{h}} = \frac{5\ \text{m}^2}{1\ \text{h}}$ or 5 m²/h.

Sam paints 5 m² every 1 h.
$90\ \text{m}^2 \div 5\ \text{m}^2 = 18$

Why do you have to divide instead of multiply?

Since 90 m² is 18 times as much as 5 m², it will take Sam 18 h to paint the entire fence.

Use a Unit Rate in h/m²

3 h to paint 15 m² can be expressed as $\frac{3\ \text{h}}{15\ \text{m}^2}$.

Determine the unit rate: $\frac{3\ \text{h}}{15\ \text{m}^2} = 0.2\ \text{h/m}^2$.

It takes Sam 0.2 h for each 1 m², so to paint the entire fence it will take him 90 times as long:
$0.2\ \text{h} \times 90 = 18\ \text{h}$.

It will take Sam 18 h to paint the entire fence.

Show You Know

Brendan is filling a backyard swimming pool with 4000 L of water. He adds 500 L of water in 20 min. How long will it take him to fill the entire pool?

Example 2: Use Proportional Reasoning With Ratios

Packaged food sold in Canada has to have a label showing the nutritional information. This label is for a package of crackers.

a) How many grams of crackers contain 15 g of fibre?

b) If you eat 75 g of crackers, how many grams of fat have you consumed?

Nutrition Facts
Serving Size 8 (28g)

Amount Per Serving	
Calories 100	
	% Daily Values*
Total Fat 5g	**8%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Sodium 180mg	**8%**
Total Carbohydrate 19g	**6%**
Dietary Fiber 4g	**16%**
Sugars 3g	
Protein 2g	**4%**

*Percent Daily Values are based on a 2,000 calorie diet.

Solution

a) ***Method 1: Use Proportional Reasoning With Words***

Write a statement that expresses the proportional relationship. Use reasoning to determine the multiplier and unknown value in each case.

"28 g of crackers contain 4 g of fibre, so how many grams of crackers will contain 15 g of fibre?

15 g ÷ 4 g is 3.75, so to have 15 g of fibre you need 3.75 servings.

3.75 servings will be 3.75 × 28 g, or 105 g of crackers."

You need 105 g of crackers to get 15 g of fibre.

Method 2: Use a Proportion Equation

Write a proportion to determine each unknown value. Solve the proportion by determining the multiplier.

$\times 3.75$

$$\frac{4\text{ g fibre}}{28\text{ g total}} = \frac{15\text{ g fibre}}{\blacksquare}$$

$\times 3.75$

How can you determine the multiplier from the proportion statement?

$28\text{ g} \times 3.75 = 105\text{ g}$

You need 105 g of crackers to get 15 g of fibre.

b) ***Method 1: Use Proportional Reasoning With Words***

"There are 5 g of fat in 28 g of crackers, so how much fat is there in 75 g of crackers?

75 g ÷ 28 g is approximately 2.68 servings.

Each serving has 5 g of fat, so 2.68 servings of crackers will have 2.68 × 5 g, or 13.4 g of fat."

If you eat 75 g of crackers you will have consumed approximately 13.4 g of fat.

Method 2: Use a Proportion Equation

Write a proportion to determine each unknown value.

Solve the proportion by determining the multiplier.

$\times 2.68$

$$\frac{5\text{ g fat}}{28\text{ g total}} = \frac{\blacksquare}{75\text{ g total}}$$

$\times 2.68$

When determining the multiplier, keep more decimal places than you want to have in the end. Why does this help make your answer more accurate?

$5\text{ g} \times 2.68 \approx 13.4\text{ g}$

Why is the answer only approximate rather than *exactly* 13.4 g?

If you eat 75 g of crackers you will consume approximately 13.4 g of fat.

Show You Know

The nutritional label shown is for bag of organic granola.

a) How many grams of cereal do you need to eat if you want to consume 12 g of protein?

b) If you eat 80 g of cereal, how much sugar have you eaten?

Nutrition Facts

Serving Size 1/2 cup (55g)

Amount Per Serving	
Calories 260	
	% Daily Values*
Total Fat 10g	**15%**
Saturated Fat 1g	**5%**
Trans Fat 0g	
Sodium 135mg	**6%**
Total Carbohydrate 37g	**12%**
Dietary Fiber 6g	**24%**
Sugars 12g	
Protein 5g	**10%**

*Percent Daily Values are based on a 2,000 calorie diet.

Example 3: Analyzing and Identifying a Relationship as Proportional

Megan and her friends organize a food drive to support their community's food bank. One day they collect 240 items in 5 hours. The next day they collect 400 items in 8 hours. Do these quantities represent a proportional relationship?

Solution

Method 1: Use a Proportion to Compare Ratios

In a proportional relationship, ratios are equivalent. Write two ratios that
- compare the number of items,
- compare the hours.

Then, use a proportion to determine if they are equivalent.

$$\frac{240 \text{ items}}{400 \text{ items}} = \frac{5 \text{ hours}}{8 \text{ hours}}$$

$$0.6 \neq 0.625$$

Why are there no units on the two decimal values here?

The ratios for the number of items and the number of hours are not equivalent, so this situation does not represent a proportional relationship.

Method 2: Use a Proportion to Compare Unit Rates

In a proportional relationship, unit rates are equivalent. Write a unit rate for number of items per hour for each day and compare them using a proportion.

$$\frac{240 \text{ items}}{5 \text{ hours}} = \frac{400 \text{ items}}{8 \text{ hours}}$$

$$48 \text{ items/h} \neq 50 \text{ items/h}$$

The rate of collection is not the same for the two days, so this situation does not represent a proportional relationship.

Show You Know

Daniel scored 20 points in the first 8 basketball games of the season. He scored 55 points in the remaining 22 games. Is this a proportional relationship?

Connect and Reflect

Key Ideas

- A proportion is a relationship in which two ratios or two rates are equivalent.
- You can use words or fractions to show proportional relationships.

30 sit-ups in 2 minutes is an equivalent rate to 90 sit-ups in 6 minutes, or $\frac{30 \text{ sit-ups}}{2 \text{ min}} = \frac{90 \text{ sit-ups}}{6 \text{ min}}$.

$$\overset{\times 3}{\frac{30 \text{ sit-ups}}{2 \text{ min}} \rightarrow \frac{90 \text{ sit-ups}}{6 \text{ min}}}$$

- You can use proportions to determine unknown values by
  - writing a proportion equation and determining the multiplier (or divisor) involved, or
  - using a unit rate.
- You can compare ratios or rates to determine whether quantities in a situation have a proportional relationship.

Practise

For help with #1 to #5, refer to Example 1 on pages 241–242.

1. Determine the value that makes each proportion statement true. Include the units. Show how you know.

a) $\frac{90 \text{ km}}{6 \text{ h}} = \frac{30 \text{ km}}{\blacksquare}$

b) $\frac{5 \text{ goals}}{2 \text{ games}} = \frac{\blacksquare}{24 \text{ games}}$

c) $\frac{12 \text{ beats}}{10 \text{ s}} = \frac{\blacksquare}{60 \text{ s}}$

2. Determine the missing value in each situation.

a) Dinner rolls are priced at 3 for 99¢. How much will 15 rolls cost?

b) Seven identical objects have a mass of 14 kg. What is the mass of 100 of these objects?

3. What is the unknown quantity in each case?

a) Two pens cost 94¢. How much will 21 pens cost?

b) A stack of 6 blocks is 24 cm high. How many blocks will it take to make a stack that is 2 m tall?

4. Matt is paid $58 for 5 h of babysitting. How much should he receive for 2 h?

5. Delia runs 300 m in 36 seconds.

a) How long will it take her to run 1000 m?

b) What assumptions did you make in your solution to part a)?

For help with #6 to #8, refer to Example 2 on pages 242–243.

6. Determine the missing value in each proportion. Show how you know.

a) $\frac{2}{3} = \frac{\blacksquare}{15}$

b) $\frac{\blacksquare}{8} = \frac{33}{24}$

c) $\frac{2}{\blacksquare} = \frac{5}{35}$

d) $\frac{20}{8} = \frac{45}{\blacksquare}$

7. A baseball player has a home run to strikeouts ratio of 3 : 17. How many home runs should he hit if he strikes out 187 times?

8. A small gear turns 18 times in the same time that a large gear turns 4 times. How many times will the large gear turn if the small gear turns 54 times?

For help with #9 and #10, refer to Example 3 on page 244.

9. Is each proportion statement below true? Show how you know.

a) $\frac{24}{15} = \frac{40}{25}$ **b)** $\frac{150 \text{ cars}}{20 \text{ min}} = \frac{525 \text{ cars}}{75 \text{ min}}$

10. Determine whether each situation represents a proportional relationship. Show how you know.

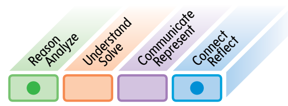
a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.

b) There are 13 boys and 15 girls in Owen's class, and there are 70 boys and 80 girls in the entire school.

c) On a map, 1 cm represents 25 km. Kendra wants to ride her bike 160 km. This distance is 6.4 cm on the map.

Apply

11. Richie and Sheena are running at the same speed around a track. Richie started first and had completed 6 laps when Sheena had completed 4 laps. Wendy and Jordan are watching.



a) Jordan says that a proportion can be used to determine how many laps Sheena will have finished when Richie has run 15 laps. Wendy says that a proportion cannot be used in this case. Who is right? Explain your thinking.

b) How many laps will Sheena have finished once Richie has completed 15 laps?

12. The student council is holding a carnival as a fundraiser. They are going to charge $10 for 3 rides on the Wild Slider. What will it cost for 18 rides on Wild Slider?

13. Determine the missing value in each equivalent fraction.

a) $\frac{6}{\blacksquare} = \frac{15}{24} = \frac{\blacksquare}{20}$

b) $\frac{48 \text{ km}}{\$30} = \frac{588 \text{ km}}{\blacksquare} = \frac{\blacksquare}{\$64}$

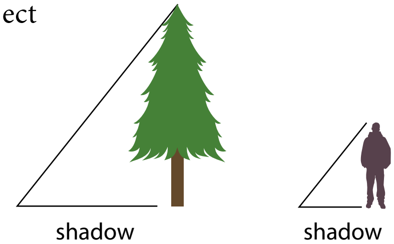
14. A breakfast cereal contains corn, wheat, and rice in the ratio of 3 to 4 to 2. If a box of cereal contains 225 g of corn, how much rice does it contain?

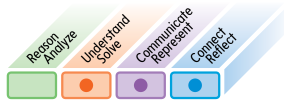
15. Reina drives 55 km from West Vancouver to Squamish in 45 minutes and then stops for lunch. After lunch she drives another 60 km from Squamish to Whistler in 50 minutes.

a) Does this represent a proportional relationship? Show how you know.

b) What would Reina's driving time from Squamish to Whistler have to be for this situation to represent a proportional relationship?

16. ✓ **Competency Check** The height of an object compared to the length of its shadow is constant for all objects at any given time.



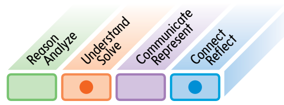


a) If a student who is 1.5 m tall casts a 1.08-m shadow, what is the height of a tree that casts a 9-m shadow?

b) Approximately how long will the shadow be for a 50-m tall tower if a 2.4-m tall post beside it casts a 1.3-m shadow?

c) Explain why this situation shows a proportional relationship.

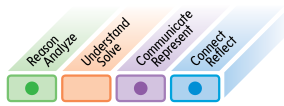
17. The dosage of medicine for a child is 2.5 mL for every 3 kg of a child's mass. What is the dose, in millilitres, for a child with a mass of 16.5 kg?



Extend

18. David can saw a log into 3 pieces in 7 min. If he continues sawing at a constant rate, how long will it take him to saw a similar log into 6 pieces?

19. A wildlife biologist wants to know how many trout are in a lake. He captures and tags 24 trout and releases them back into the lake. Two weeks later he returns and captures 30 trout and finds that 5 of them are tagged. He uses the following ratios to estimate the number of fish in the lake:



$$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish caught and tagged}}{\text{total fish in lake}}$$

a) How many fish does he estimate are in the lake?

b) Why can a proportion be used in this situation? What assumptions are being made?

20. Simone estimates that frogs eat 6 insects per hour and that dragonflies eat 9 insects per hour. He also assumes that frogs rest for 8 h each day and dragonflies rest for 13 h each day.

a) Determine the daily rate of insects eaten by a frog and a dragonfly. Which one eats more insects per day?

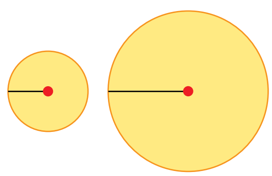
b) How many insects would a dragonfly eat in a week?

c) How many insects would a frog eat in August?

21. A circle has a radius that is twice that of another circle.

a) What is the ratio of their circumferences?

b) What is the ratio of their areas?



Rich Problems

1. The table shows the growth of some vegetables in the school garden over the course of five days.

Vegetable	Day 1	Day 2	Day 3	Day 4	Day 5
Tomato	0.5 in.	1.0 in.	1.5 in.	2.0 in.	2.5 in.
Carrot	3.9 cm	4.5 cm	5.1 cm	5.7 cm	6.3 cm
Onion	4.1 cm	5.2 cm	6.4 cm	7 cm	8.5 cm
Zucchini	2.25 cm	4.5 cm	6.75 cm	9 cm	11.25 cm

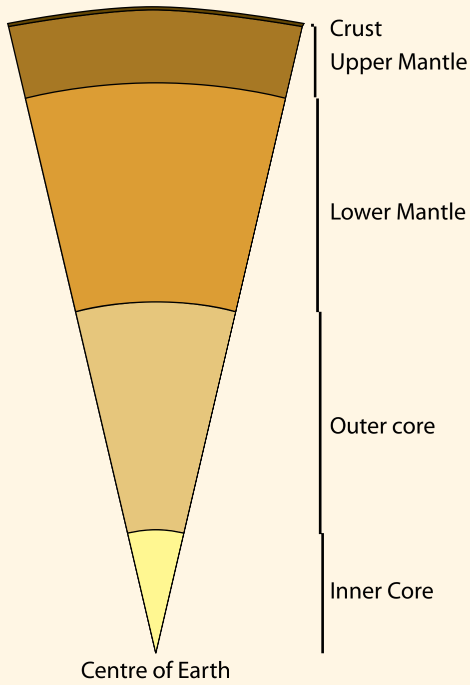
Are the relationships in the table proportional? Justify your reasoning.

2. Caleb, Jamal, and Mina are shopping for shoes. One of their favourite stores is having a "buy two pairs, get one pair free" sale. Caleb chooses a pair of basketball shoes for \$140, Jamal picks a pair of skate shoes for \$110, and Mina settles on a pair of casual flat shoes for \$90. The total cost is \$250. How much should each friend pay? Try to find the fairest way possible. Justify your reasoning.

3. The Earth is made up of 5 main layers as shown in the table.

Layer	Thickness (km)
Crust	30
Upper mantle	720
Lower mantle	2171
Outer core	2259
Inner core	1221

If Earth were the size of a basketball, how thick would each layer be? The diameter of a basketball is 24.26 centimetres.



4. A salesperson achieves 110% of her previous year's sales each year. Assume this continues. Show how many years it will take her to double her sales.

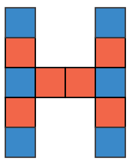
Chapter 7 Review

Learning Goals

Inquire and Explore:	**How can two quantities be compared, represented, and communicated?** **How are fractions and ratios interrelated?** **How does ratio use in mechanics differ from ratio use in architecture?**
After this section, I can	
7.1	■ represent two-term and three-term ratios in a variety of ways ■ represent part-to-part and part-to-whole ratios in a variety of ways ■ identify and describe ratios used in real life ■ analyze situations and solve problems using ratios
7.2	■ represent rates in a variety of ways ■ identify, describe, and record rates used in real life ■ use rates to analyze situations and solve problems
7.3	■ represent proportional relationships in a variety of ways ■ use proportional reasoning to determine unknown values ■ analyze situations to determine whether a proportional relationship exists

7.1 Ratios, pages 222–231

1. Use the square tile pattern to find each of the following:



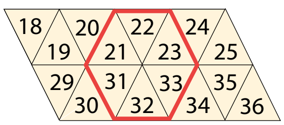
a) ratio of red squares to blue squares

b) ratio of blue squares to total squares

c) two equivalent ratios for the answer in part b)

d) percent of squares that are red

2. Look at the figure.



a) What is the ratio of two-digit numbers in the red hexagon compared to the total number of two-digit numbers?

b) Express the answer in part a) as a fraction in lowest terms.

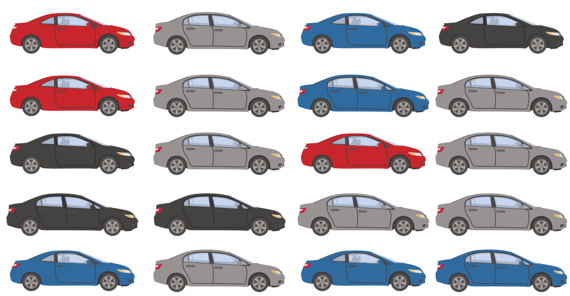
c) What is the ratio of two-digit numbers containing a 2 compared to the number of two-digit numbers in the red hexagon that contain a 2?

3. In a class of 32 students, there are 24 girls.

a) What is the ratio of boys to total students?

b) What is the ratio of girls to boys?

4. Stephanie looks at the colour of cars in a parking lot. She finds that 8 are silver, 5 are blue, 3 are red, and 4 are black.



a) What ratio does 2 : 5 represent?

b) Earlier, Stephanie had predicted that at most 30% of the cars in the lot would be silver. Was she correct?

c) If another 120-car parking lot has cars in the same colour ratio, how many cars in that lot are blue?

7.2 Rates, pages 232–239

5. Kayla usually makes her own salad dressing using oil, lemon juice, and vinegar.

a) One evening she uses 100 mL of oil, 40 mL of lemon juice, and 60 mL of vinegar. Write this as a ratio in lowest terms.

b) Another day she needs to make 700 mL of the same dressing for a salad she is bringing to a party. How much of each ingredient will she need?

6. Determine a unit rate in each situation.

a) Steven runs up 300 steps in 6 min.

b) $3.60 is the price of 4 L of milk.

c) A jet travels 2184 km in 3.5 h.

d) A polar bear gains 450 kg in 9 years.

7. The table compares the typical monthly cost of the electricity for several appliances. Which appliance has the lowest unit cost of electricity consumption?

Appliance	Time On (h)	Monthly Cost ($)
Fridge	240	17.16
Computer and monitor	120	2.73
Television	180	2.86
Treadmill	15	3.99

8. Shannon buys 12 granola bars for $9.96.

a) Determine the price per bar.

b) Explain whether your answer in part a) is a ratio or a rate.

9. Shelly rides her mountain bike at a rate of 30 km/h for 2.5 h. Josh rides his mountain bike at a rate of 35 km/h for 1 h and then slows down to 25 km/h for 1.5 h.

a) Who travels farther in 2.5 h?

b) What is the difference in the distance travelled by the cyclists?

7.3 Using Proportional Reasoning, pages 240–247

10. Determine the missing value if each rate is equivalent. Give the unit for each.

a) $\dfrac{\blacksquare}{1 \text{ month}} = \dfrac{64 \text{ kg}}{4 \text{ months}}$

b) $\dfrac{\$84}{800 \text{ km}} = \dfrac{\blacksquare}{100 \text{ km}}$

c) $\dfrac{80 \text{ beats}}{2 \text{ min}} = \dfrac{720 \text{ beats}}{\blacksquare}$

11. a) Three bars of soap cost $2.94. What is the cost of 8 bars of soap?

b) On a map, 1 cm represents 150 km. How many centimetres represent a distance of 800 km?

12. One day Minji made 24 jars of jam in 5 hours. The next day she made 60 jars of jam in 12 hours. Does this situation represent a proportional relationship? Show how you know.

13. One night 30 cm of snow fell in 6 h. The next day 63 cm of snow fell in 12 hours. Does this represent a proportional relationship? Show how you know.

14. Shawn buys 2 kg of sockeye salmon at a local market for $44.60. Maria buys 850 g of the same type of salmon. How much can she expect to pay?

15. A gardener takes a half hour to mow and weed a lawn area that measures 20 m by 15 m. She charges $25 per hour. How much should the gardener receive for a lawn area that measures 40 m by 30 m?

Connect the Concepts

16. How is a proportion different from a ratio and a rate? Explain using your own examples.

17. The values in the table show the cost a company charges for printing custom T-shirts.

Number of Shirts	Cost ($)
10	200
15	250
20	300
25	350
30	400

a) Does the relationship of the cost to the number of shirts represent a ratio or a rate? Explain.

b) What is the unit cost if a customer places an order for 25 shirts?

c) Does this situation represent a proportional relationship? Justify your answer mathematically.

18. At the beginning of the season, a football team won 5 games and lost 3 games.

a) The coach had predicted that they would win 60% of their games. Have they done that so far?

b) They have scored 135 points so far. At what rate are they scoring, in points per game?

c) If they keep scoring at this same rate, how many points will they score in total over the 18-game season?

d) By the end of the season they had won 11 games. Does this represent a proportional relationship with their results from the first part of the season?

e) Was the coach's prediction accurate? Explain.