1 Chapter Review

FREQUENTLY ASKED Questions

Q: Why is it important to check carefully any proof you develop and the associated reasoning you used?

A: If you have made a statement using flawed reasoning, you can end up with a conclusion that does not make sense. For example:
All pitbulls are ferocious dogs.
Jake is a pitbull.
Jake is a ferocious dog.

In this example, the premise is faulty. Pitbulls are known to be a ferocious breed, but we cannot be certain that every pitbull in the world is a ferocious dog. This leads to the invalid conclusion that Jake is ferocious.

Study *Aid*

- See Lesson 1.5, Examples 1 and 2.
- Try Chapter Review Questions 11 and 12.

Q: What can deductive reasoning and inductive reasoning be used for?

A1: Inductive reasoning can help you make conjectures. You can look for patterns in several examples or cases. The patterns you observe in specific cases can be generalized to a statement that includes all cases. This forms the basis for a conjecture. Deductive reasoning gives you the ability to prove that your conjecture is valid for every possible case.

For example, when you examine the sum of three consecutive natural numbers, you can see a pattern.
$1 + 2 + 3 = 6$
$2 + 3 + 4 = 9$
$3 + 4 + 5 = 12$
$4 + 5 + 6 = 15$

Each sum is a multiple of 3. Based on this pattern, you can make the following conjecture: The sum of three consecutive natural numbers is divisible by 3. You can then use deductive reasoning to prove your conjecture.

x, $x + 1$, $x + 2$	Let x, $x + 1$, and $x + 2$ represent three consecutive natural numbers.
$x + x + 1 + x + 2$	Write an expression for the sum.
$3x + 3$	Collect like terms.
$3(x + 1)$	Factor.
$3(x + 1)$ is divisible by 3.	Since 3 is a factor of the expression, the expression will always be divisible by 3.

Study Aid

- See Lesson 1.6, Examples 1 and 2, and Lesson 1.7, Examples 1 and 2.
- Try Chapter Review, Questions 12 to 15.

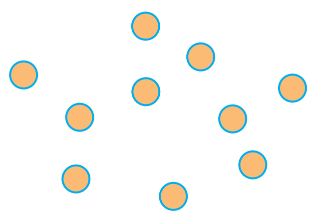
A2: Inductive and deductive reasoning are useful strategies for solving problems, some types of puzzles, and some types of games.

Word puzzle: Identify a five-letter word correctly in five or fewer attempts. After each attempt, the person who knows the word will show you the correct letters in the correct positions in yellow, the correct letters in the wrong positions in red, and the incorrect letters with no colour.

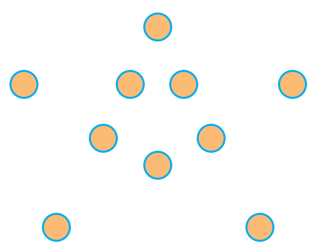
	Response
My first guess is RENTS, since it has letters that are often used in English words.	RENTS
I know that S is in the correct position. R is in the word but in a different position. E, N, and T are not in the word. The letter R is often used after the letters C, D, F, G, and P, so that could put R in the second position. I chose DRAWS as my next guess, because A is the next most common vowel.	DRAWS
I know that the vowel has to be I, O, or U. I also know that the consonants D, R, and W are in the word, but in the wrong positions in DRAWS. From my guesses, I think that R goes in the third position since I can't think of a word that would have either _WDRS or _DWRS. Also, from the patterns of letters, I think the R and D go together. That would make the unknown word to be W_RDS. The only vowel that would fit is O. My guess is WORDS.	WORDS

Problem: Make five lines of counters, with four counters in each line, using only 10 counters.

Since an array of (5)(4) counters would require 20 counters, I deduce that most counters must be part of more than one line. This means that the lines must overlap.



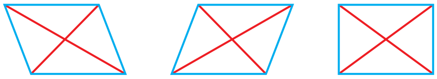
I can also deduce that the lines need to overlap more than once since there are only 10 counters available. Lines of four counters could overlap at any counter.



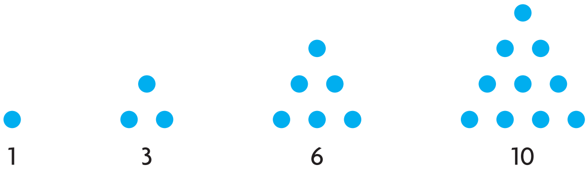
PRACTISING

Lesson 1.1

1. Charles studied the diagonals of the parallelograms below to look for patterns. Make a conjecture about the diagonals of parallelograms. What evidence supports your conjecture?



2. Consider the following sequence of triangular numbers:



a) Describe the pattern, and use it to determine the next four triangular numbers.

b) Consider the products $1 \cdot 2$, $2 \cdot 3$, $3 \cdot 4$, $4 \cdot 5$. Explain how these products are related to each triangular number.

c) Make a conjecture about a formula you could use to determine the *n*th triangular number.

3. Examine the following number pattern:

$1^3 = 1$ and $1 = 1^2$

$1^3 + 2^3 = 9$ and $9 = 3^2$

$1^3 + 2^3 + 3^3 = 36$ and $36 = 6^2$

$1^3 + 2^3 + 3^3 + 4^3 = 100$ and $100 = 10^2$

a) Describe the pattern you see.

b) Use your observations to predict the next equation in the pattern.

c) Make a conjecture about the sum of the first n cubes.

Lesson 1.2

4. a) Examine this pattern to determine the next equation:

$37 \times 3 = 111$

$37 \times 6 = 222$

$37 \times 9 = 333$

$37 \times 12 = 444$

b) Is your conjecture correct? Explain how you know.

c) This pattern eventually breaks down. Determine when the breakdown occurs.

Lesson 1.3

5. a) What is a counterexample?

b) Explain why counterexamples are useful.

6. Harry claims that if opposite sides of a quadrilateral are the same length, the quadrilateral is a rectangle. Do you agree or disagree? Justify your decision.

7. Sadie claims that the difference between any two positive integers is always a positive integer. Do you agree or disagree? Justify your decision.

Lesson 1.4

8. Complete the conclusion for the following deductive argument: If an integer is an even number, then its square is also even. Six is an even number, therefore, ….

9. Prove that the product of two odd integers is always odd.

10. Linda came across this number trick on the Internet and tried it:

- Think about the date of your birthday.
- Multiply the number for the month of your birthday by 5. (For example, the number for November is 11.)
- Add 7.
- Multiply by 4.
- Add 13.
- Multiply by 5.
- Add the day of your birthday.
- Subtract 205.
- Write your answer.

a) Try the trick. What did you discover?

b) Prove how this trick works. Let m represent the number for the month of your birthday and d represent the day.

11. Examine the relationships below.

$2(3^2 + 5^2) = 2^2 + 8^2$
$2(2^2 + 3^2) = 1^2 + 5^2$
$2(7^2 + 4^2) = 3^2 + 11^2$

a) Describe the patterns you see.

b) Jen makes the following conjecture: If you double the sum of two squares, the product is always the sum of two squares. Prove Jen's conjecture.

Lesson 1.5

12. The following proof seems to show that $2 = 1$. Examine this proof, and determine where the error in reasoning occurred.

Let $a = b$.

$$\begin{aligned} a^2 &= ab && \text{Multiply by } a. \\ a^2 - b^2 &= ab - b^2 && \text{Subtract } b^2. \\ (a-b)(a+b) &= b(a-b) && \text{Factor.} \\ a + b &= b && \text{Divide by } (a-b). \\ b + b &= b && a = b \\ 2b &= b && \text{Simplify.} \\ 2 &= 1 && \text{Divide by } b. \end{aligned}$$

13. Julie was trying to prove that a number trick always results in 5:

Julie's Proof

n	Choose a number.
$n + 10$	Add 10.
$5n + 10$	Multiply the total by 5.
$5n - 40$	Subtract 50.
$\dfrac{5n - 40}{n}$	Divide by the number you started with.

Identify the error in Julie's proof, and correct it.

Lesson 1.6

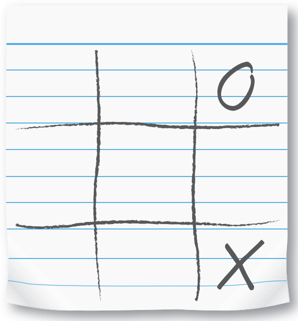
14. Two mothers and two daughters got off a city bus, reducing the number of passengers by three. Explain how this is possible.

15. The three little pigs built three houses: one of straw, one of sticks, and one of bricks. By reading the six clues, deduce which pig built each house, the size of each house, and the town in which each house was located.

Clues

- Penny Pig did not build a brick house.
- The straw house was not medium in size.
- Peter Pig's house was made of sticks, and it was neither medium nor small in size.
- Patricia Pig built her house in Pleasantville.
- The house in Hillsdale was large.
- One house was in a town called Riverview.

Lesson 1.7



16. If you are playing next in this game of tic-tac-toe, you can use a strategy that guarantees you will win the game. Explain what this strategy is.

17. The rules for the game of 15 are given below:

- The cards are placed on a table between two players.
- Players take turns choosing a card (any card they like).
- The winner is the first player to have three cards that add to 15.
  For example, if you drew 1, 5, 6 and 8, then you would win, because $1 + 6 + 8 = 15$.

a) Is it possible to win the game in three moves?

b) Devise a winning strategy. Explain your strategy.