2.3 Angle Properties in Triangles

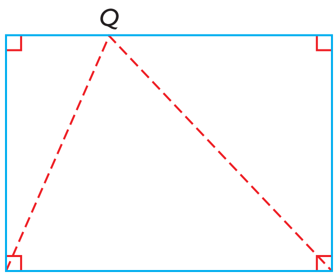
YOU WILL NEED

- dynamic geometry software OR compass, protractor, and ruler
- scissors

EXPLORE...

On a rectangular piece of paper, draw lines from two vertices to a point on the opposite side. Cut along the lines to create two right triangles and an acute triangle.

- What do you notice about the three triangles?
- Can you use angle relationships to show that the sum of the measures of the angles in any acute triangle formed this way is 180°?

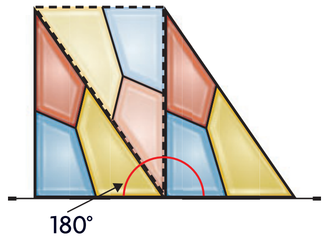


GOAL

Prove properties of angles in triangles, and use these properties to solve problems.

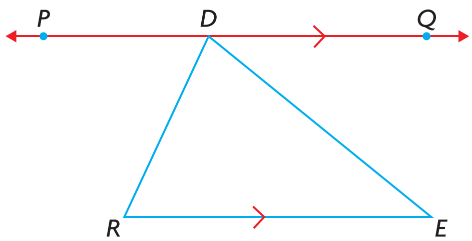
INVESTIGATE the Math

Diko placed three congruent triangular tiles so that a different angle from each triangle met at the same point. She noticed the angles seemed to form a straight line.



? Can you prove that the sum of the measures of the interior angles of any triangle is 180°?

A. Draw an acute triangle, $\triangle RED$. Construct line PQ through vertex D, parallel to RE.



B. Identify pairs of equal angles in your diagram. Explain how you know that the measures of the angles in each pair are equal.

C. What is the sum of the measures of $\angle PDR$, $\angle RDE$, and $\angle QDE$? Explain how you know.

D. Explain why:
$\angle DRE + \angle RDE + \angle RED = 180°$

E. In part A, does it matter which vertex you drew the parallel line through? Explain, using examples.

F. Repeat parts A to E, first for an obtuse triangle and then for a right triangle. Are your results the same as they were for the acute triangle?

Reflecting

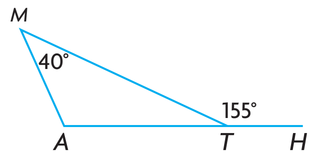
G. Why is Diko's approach not considered to be a proof?

H. Are your results sufficient to prove that the sum of the measures of the angles in any triangle is 180°? Explain.

APPLY the Math

EXAMPLE 1 Using angle sums to determine angle measures

In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



Serge's Solution

$$\angle MTA + \angle MTH = 180°$$
$$\angle MTA + (155°) = 180°$$
$$\angle MTA = 25°$$

$\angle MTA$ and $\angle MTH$ are supplementary since they form a straight line.

$$\angle MAT + \angle AMT + \angle MTA = 180°$$
$$\angle MAT + (40°) + (25°) = 180°$$
$$\angle MAT = 115°$$

The sum of the measures of the interior angles of any triangle is 180°.

The measures of the unknown angles are:
$\angle MTA = 25°$; $\angle MAT = 115°$.

Your Turn

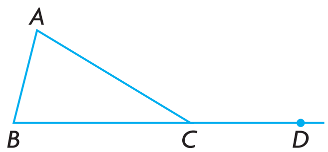
If you are given one interior angle and one exterior angle of a triangle, can you always determine the other interior angles of the triangle? Explain, using diagrams.

EXAMPLE 2 Using reasoning to determine the relationship between the exterior and interior angles of a triangle

Determine the relationship between an exterior angle of a triangle and its **non-adjacent interior angles**.

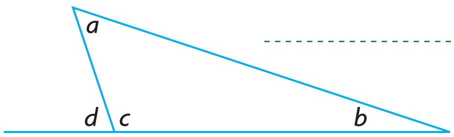
non-adjacent interior angles

The two angles of a triangle that do not have the same vertex as an exterior angle.



$\angle A$ and $\angle B$ are non-adjacent interior angles to exterior $\angle ACD$.

Joanna's Solution



I drew a diagram of a triangle with one exterior angle. I labelled the angle measures a, b, c, and d.

$\angle d + \angle c = 180°$

$\angle d = 180° - \angle c$

$\angle d$ and $\angle c$ are supplementary. I rearranged these angles to isolate $\angle d$.

$\angle a + \angle b + \angle c = 180°$

$\angle a + \angle b = 180° - \angle c$

The sum of the measures of the angles in any triangle is 180°.

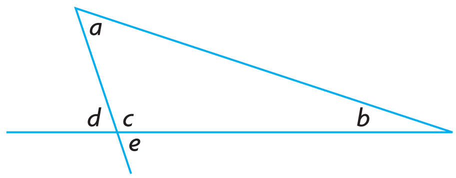
$\angle d = \angle a + \angle b$

Since $\angle d$ and $(\angle a + \angle b)$ are both equal to $180° - \angle c$, by the transitive property, they must be equal to each other.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

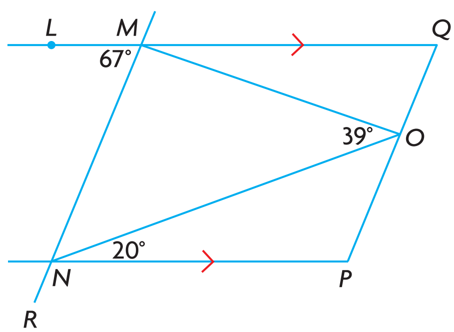
Your Turn

Prove: $\angle e = \angle a + \angle b$



EXAMPLE 3 Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



Tyler's Solution

MN is a transversal of parallel lines LQ and NP.

MN intersects parallel lines LQ and NP.

$$\angle MNO + 20° = 67°$$
$$\angle MNO = 47°$$

Since $\angle LMN$ and $\angle MNP$ are alternate interior angles between parallel lines, they are equal.

$$\angle NMO + \angle MNO + 39° = 180°$$
$$\angle NMO + (47°) + 39° = 180°$$
$$\angle NMO + 86° = 180°$$
$$\angle NMO = 94°$$

The measures of the angles in a triangle add to 180°.

$$\angle NMO + \angle QMO + 67° = 180°$$
$$(94°) + \angle QMO + 67° = 180°$$
$$161° + \angle QMO = 180°$$
$$\angle QMO = 19°$$

$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so their measures must add to 180°.

The measures of the angles are:
$\angle MNO = 47°$; $\angle NMO = 94°$; $\angle QMO = 19°$.

Dominique's Solution

$$\angle NMO + \angle MNO + 39° = 180°$$
$$\angle NMO + \angle MNO = 141°$$

The sum of the measures of the angles in a triangle is 180°.

$$(\angle NMO + \angle QMO) + (\angle MNO + 20°) = 180°$$
$$\angle NMO + \angle MNO + \angle QMO = 160°$$

The angles that are formed by $(\angle NMO + \angle QMO)$ and $(\angle MNO + 20°)$ are interior angles on the same side of transversal MN. Since $LQ \parallel NP$, these angles are supplementary.

$$(141°) + \angle QMO = 160°$$
$$\angle QMO = 19°$$

I substituted the value of $\angle NMO + \angle MNO$ into the equation.

$$\angle NMO + \angle QMO + 67° = 180°$$
$$\angle NMO + (19°) + 67° = 180°$$
$$\angle NMO = 94°$$

$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so the sum of their measures is 180°.

$$\angle NMO + \angle MNO = 141°$$
$$(94°) + \angle MNO = 141°$$
$$\angle MNO = 47°$$

The measures of the angles are:
$\angle QMO = 19°$; $\angle NMO = 94°$; $\angle MNO = 47°$.

Your Turn

In the diagram for Example 3, $QP \parallel MR$. Determine the measures of $\angle MQO$, $\angle MOQ$, $\angle NOP$, $\angle OPN$, and $\angle RNP$.

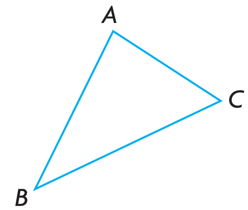
In Summary

Key Idea

- You can prove properties of angles in triangles using other properties that have already been proven.

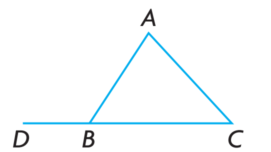
Need to Know

- In any triangle, the sum of the measures of the interior angles is proven to be 180°.



$\angle A + \angle B + \angle C = 180°$

- The measure of any exterior angle of a triangle is proven to be equal to the sum of the measures of the two non-adjacent interior angles.



$\angle DBA = \angle BAC + \angle ACB$

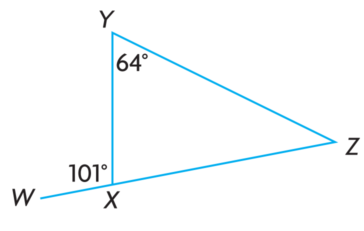
CHECK Your Understanding

1. Harrison drew a triangle and then measured the three interior angles. When he added the measures of these angles, the sum was 180°. Does this prove that the sum of the measures of the angles in any triangle is 180°? Explain.

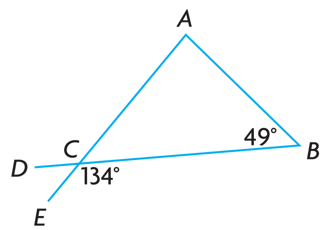
2. Marcel says that it is possible to draw a triangle with two right angles. Do you agree? Explain why or why not.

3. Determine the following unknown angles.

a) $\angle YXZ, \angle Z$

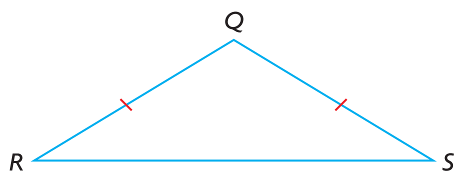


b) $\angle A, \angle DCE$

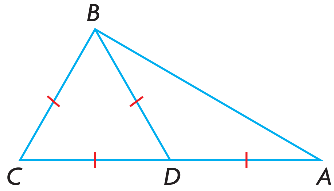


PRACTISING

4. If $\angle Q$ is known, write an expression for the measure of one of the other two angles.

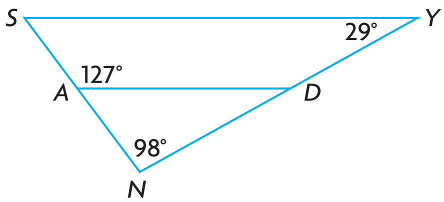


5. Prove: $\angle A = 30°$



6. Determine the measures of the exterior angles of an equilateral triangle.

7. Prove: $SY \parallel AD$

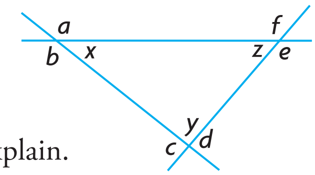


8. Each vertex of a triangle has two exterior angles, as shown.

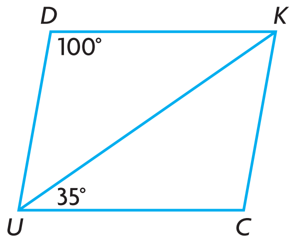
a) Make a conjecture about the sum of the measures of $\angle a$, $\angle c$, and $\angle e$.

b) Does your conjecture also apply to the sum of the measures of $\angle b$, $\angle d$, and $\angle f$? Explain.

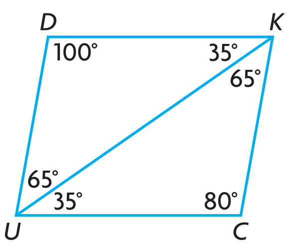
c) Prove or disprove your conjecture.



9. *DUCK* is a parallelogram. Benji determined the measures of the unknown angles in *DUCK*. Paula says he has made an error.



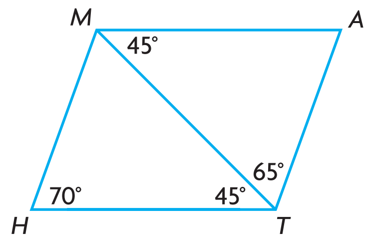
Benji's Solution

Statement	Justification
$\angle DKU = \angle KUC$	$\angle DKU$ and $\angle KUC$ are alternate interior angles.
$\angle DKU = 35°$	
$\angle UDK = \angle DUC$	$\angle UDK$ and $\angle DUC$ are corresponding angles.
$\angle DUK + \angle KUC = 100°$	$\angle DUK$ and $\angle UKC$ are alternate interior angles.
$\angle DUK = 65°$	
$\angle UKC = 65°$	
$\angle UCK = 180° - (\angle KUC + \angle UKC)$	The sum of the measures of the angles in a triangle is 180°.
$\angle UCK = 180° - (35° + 65°)$	
$\angle UCK = 80°$	
	I redrew the diagram, including the angle measures I determined.

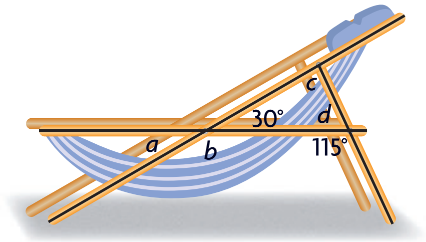
a) Explain how you know that Benji made an error.

b) Correct Benji's solution.

10. Prove that quadrilateral *MATH* is a parallelogram.

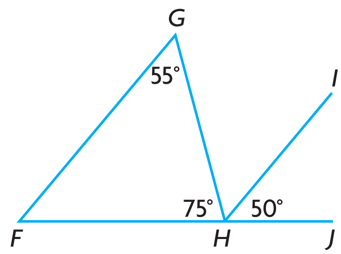


11. A manufacturer is designing a reclining lawn chair, as shown. Determine the measures of $\angle a$, $\angle b$, $\angle c$, and $\angle d$.

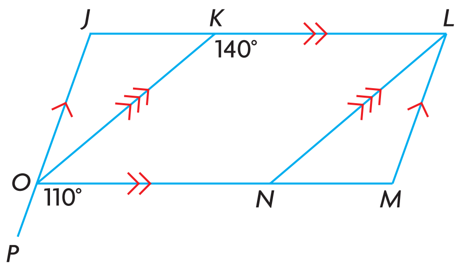


12. a) Tim claims that *FG* is not parallel to *HI* because $\angle FGH \neq \angle IHJ$. Do you agree or disagree? Justify your decision.

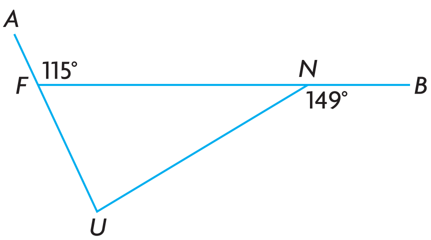
b) How else could you justify your decision? Explain.



13. Use the given information to determine the measures of $\angle J$, $\angle JKO$, $\angle JOK$, $\angle KLM$, $\angle KLN$, $\angle M$, $\angle LNO$, $\angle LNM$, $\angle MLN$, $\angle NOK$, and $\angle JON$.

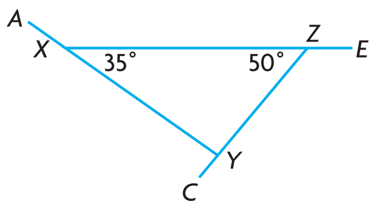


14. Determine the measures of the interior angles of $\triangle FUN$.

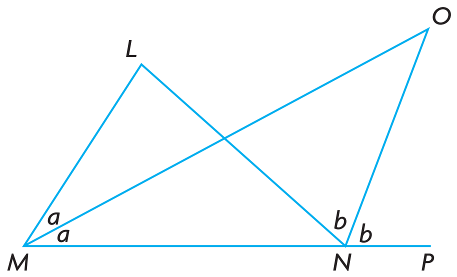


15. a) Determine the measures of $\angle AXZ$, $\angle XYC$, and $\angle EZY$.

b) Determine the sum of these three exterior angles.



16. MO and NO are angle bisectors.
Prove: $\angle L = 2\angle O$



Closing

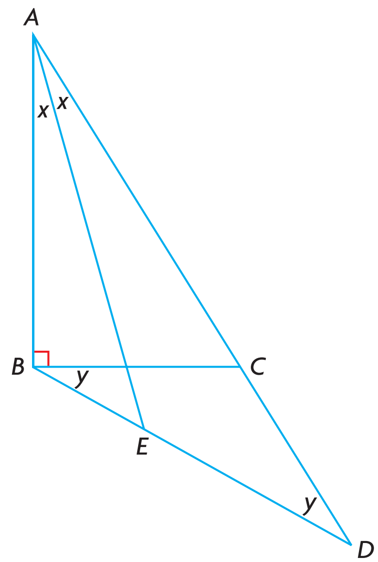
17. Explain how drawing a line that is parallel to one side of any triangle can help you prove that the sum of the angles in the triangle is 180°.

Extending

18. Given: AE bisects $\angle BAC$.

$\triangle BCD$ is isosceles.

Prove: $\angle AEB = 45°$



19. $\triangle LMN$ is an isosceles triangle in which $LM = LN$. ML is extended to point D, forming an exterior angle, $\angle DLN$. If $LR \parallel MN$, where N and R are on the same side of MD, prove that $\angle DLR = \angle RLN$.